

Indeterminacy of Quantum Geometry

Craig Hogan

Fermilab and U. Chicago

What is the smallest interval of time?

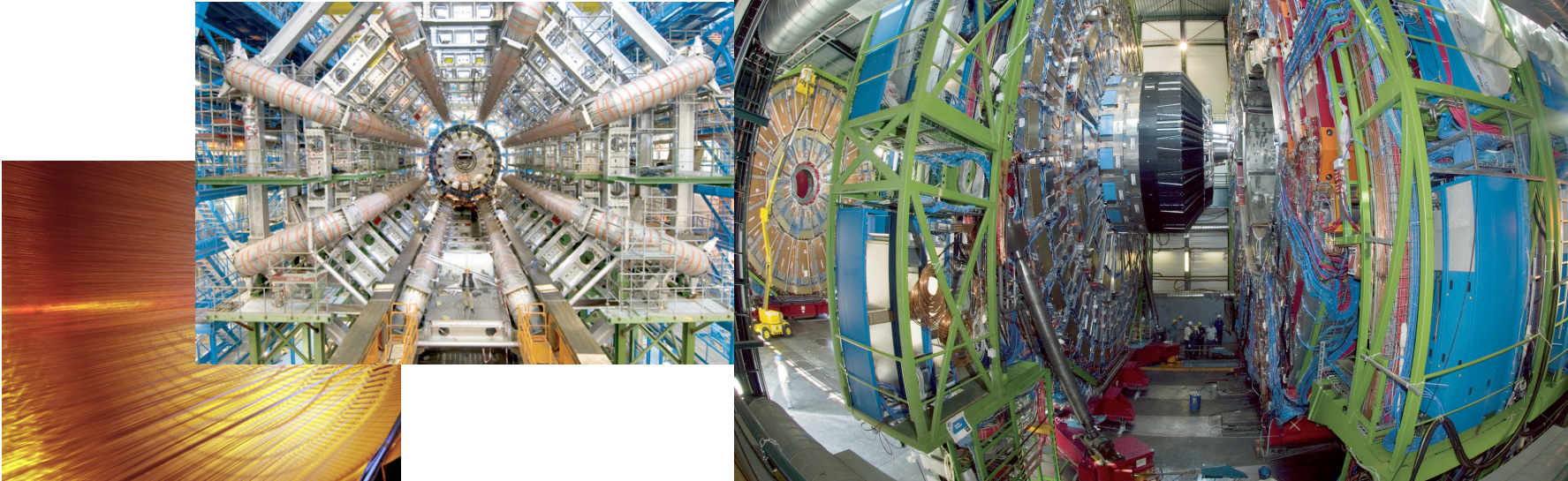
- Quantum gravity suggests a minimum (Planck) time,

$$t_P \equiv l_P/c \equiv \sqrt{\hbar G_N/c^5} = 5 \times 10^{-44} \text{ seconds}$$

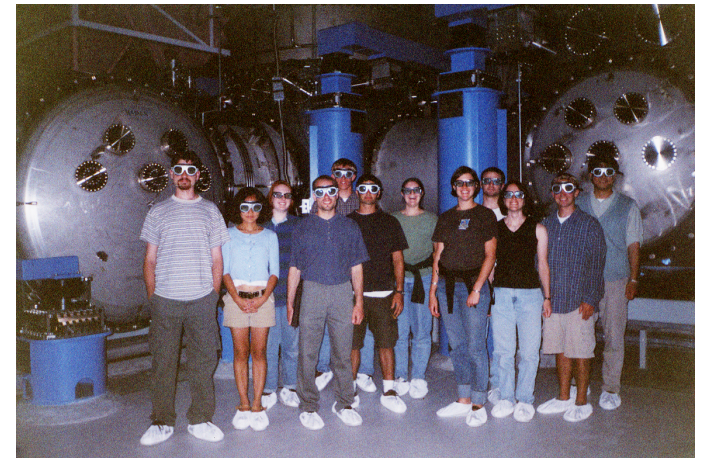
- How can it be measured directly?
- \sim particle energy 10^{16} TeV

What is the best microscope for measuring quantum geometry?

CERN/FNAL: $\text{TeV}^{-1} \sim 10^{-18} \text{ m}$



LIGO/GEO600: $\sim 10^{-19} \text{ m}$ over
 $\sim 10^3 \text{ m}$ baseline



Ways to study quantum physics of empty space

- Dark Energy is a phenomenon of empty space and gravitation
- Observed via effect on cosmic expansion and structure
- New idea: Quantum physics of empty space may be studied directly in the lab using interferometers
- Not gravitational waves



vibration-isolated platform



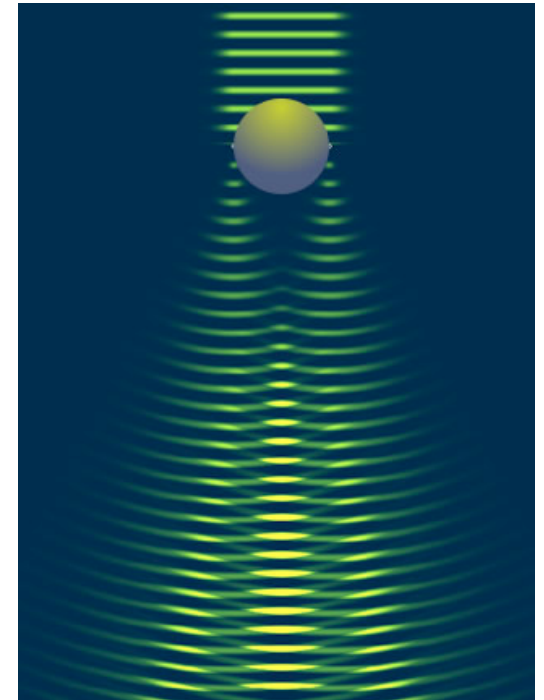
initial alignment



test mass suspended on fine wire

Holographic Quantum Geometry

- Spacetime is a quantum system, not a continuous classical manifold
- Quantum theory in 2+1 dimensions
- Positions are wavefunctions
- Spacetime state shaped by measurement: choice of light sheet or wavefront direction
- Planck frequency= maximum possible
- Transverse wavefunction spreads over macroscopic distances
- transverse indeterminacy in geometry much larger than Planck length



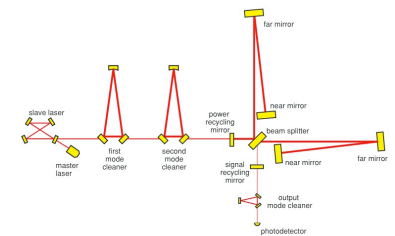
$$l_P = \sqrt{\hbar G_N / c^3} = 1.616 \times 10^{-33} \text{cm}$$

Position uncertainty in holographic quantum geometry

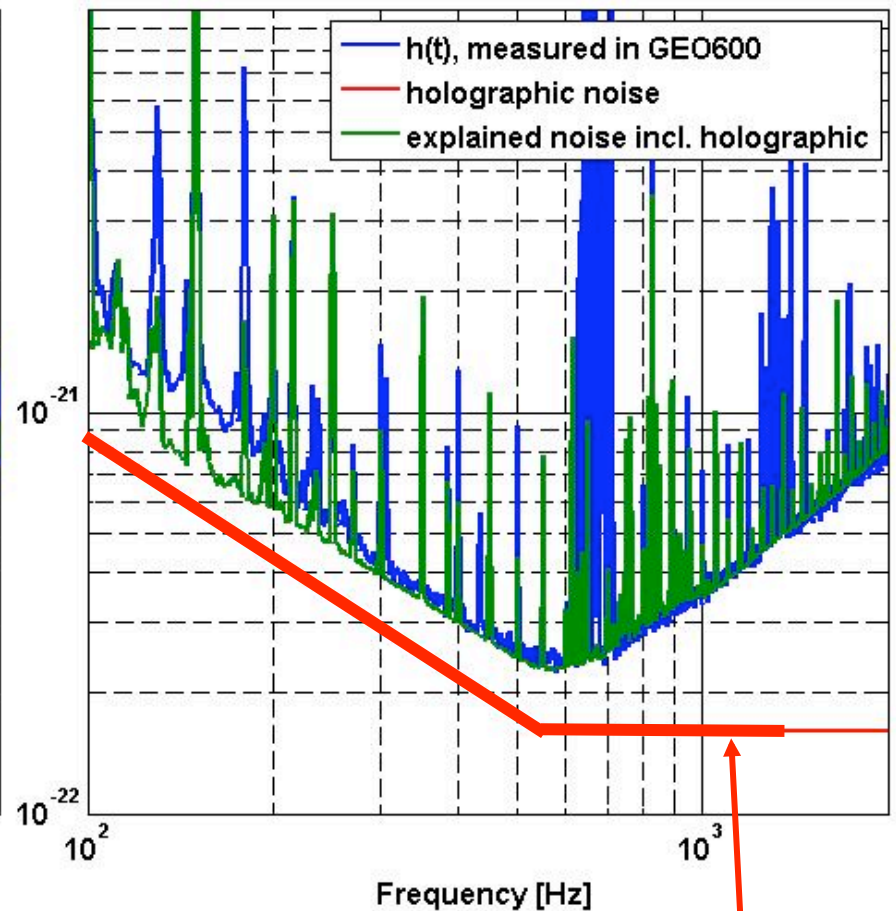
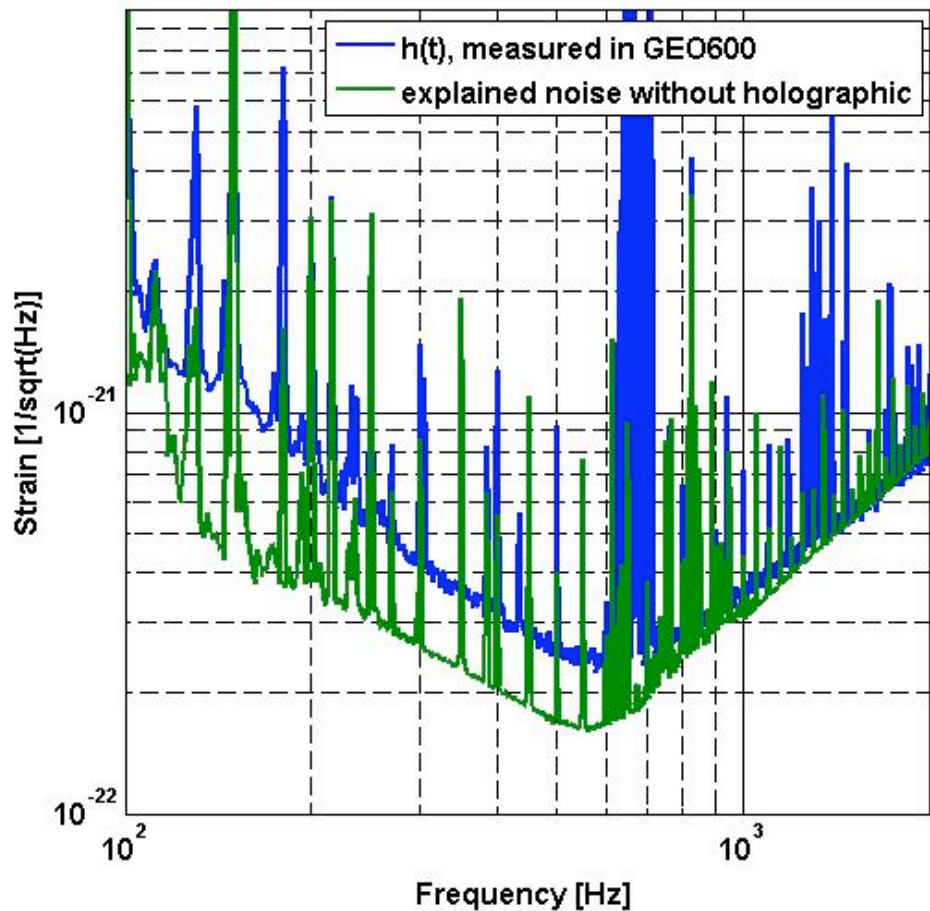
- Standard quantum field theory: geometry is classical, fields are quantized, modes are independent, "all physics is local"
- Gravitation theory suggests holographic quantum geometry on 2D light sheets, UV cutoff at the Planck length; 3D states and correlations are spatially nonlocal in "virtual" z, t dimension
- Theory of position based on wave optics: position wavefunctions represented by field correlations transverse to light sheet
- New holographic phenomenology: x, y positions of macroscopic bodies are indeterminate by the geometric mean of the Planck length and their z, t separation

Direct measurement of quantum geometry fluctuations

- Experimental approach to small lengths/large energies has been the high energy frontier-- accelerators
- New: ultraprecise coherent macroscopic position measurements by interferometers designed for gravitational wave detection ("Standard Quantum Limit": small Heisenberg uncertainty)
- Holographic quantum geometry predicts a new detectable effect: "holographic noise"
- black hole evaporation physics--- in the lab
- Spectrum and distinctive spatial character of the noise is predicted with no parameters, and may already be detected
- An experimental program is motivated
- CJH: [arXiv:0806.0665](https://arxiv.org/abs/0806.0665), also PRD D 77, 104031 (2008)



“Mystery Noise” in GEO600



Data: S. Hild (GEO600)

Prediction: [arXiv:0806.0665](https://arxiv.org/abs/0806.0665)

Total noise: not fitted

zero-parameter prediction
for holographic noise in
GEO600 (~ equivalent
strain)

Holographic Quantum Geometry: theory

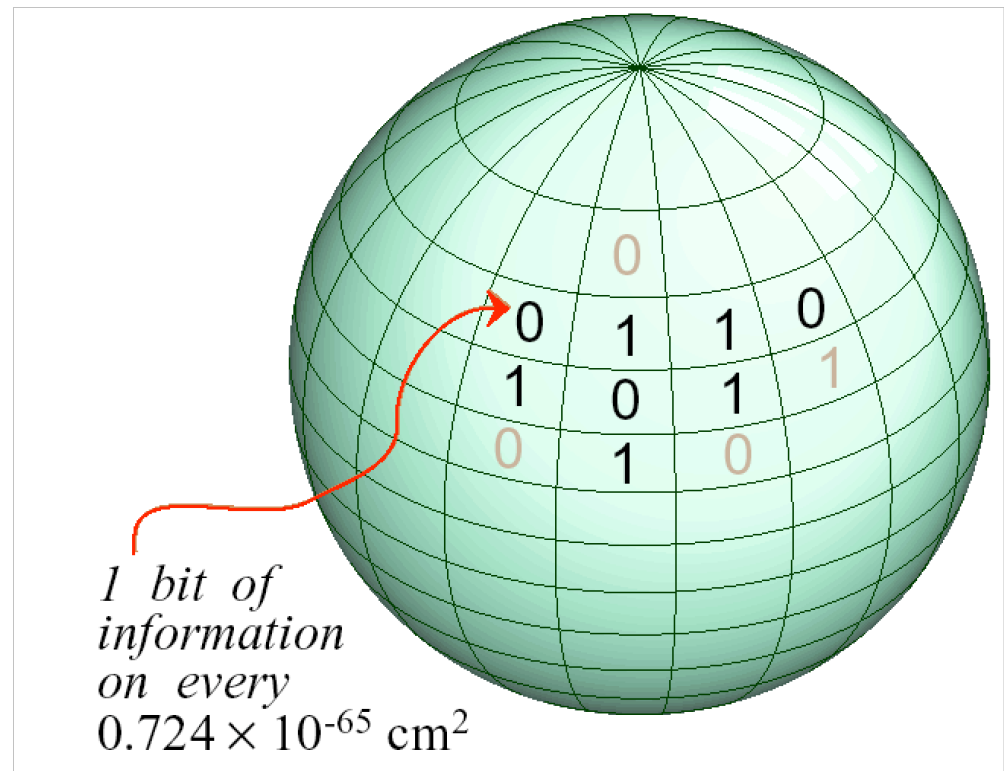
- Black holes: entropy=area/4 $S = A/l_P^2 4 \ln 2$
- Black hole evaporation
- Einstein's equations from heat flow
- Classical GR from surface theory
- Universal covariant entropy bound
- Exact state counts of extremal holes in large D
- AdS/CFT type dualities: N-1 dimensional duals
- M theory: noncommuting transverse positions
- All suggest that quantum geometry lives on 2+1 dimensional null surfaces

Beckenstein, Hawking, Bardeen et al.,
'tHooft, Susskind, Bousso, Srednicki,
Jacobson, Padmanabhan, Banks,
Fischler, Shenker, Unruh

“This is what we found out about Nature’s book keeping system: the data can be written onto a surface, and the pen with which the data are written has a finite size.”

-Gerard ‘t Hooft

Everything about the 3D world can be encoded on a 2D surface at Planck resolution (?)

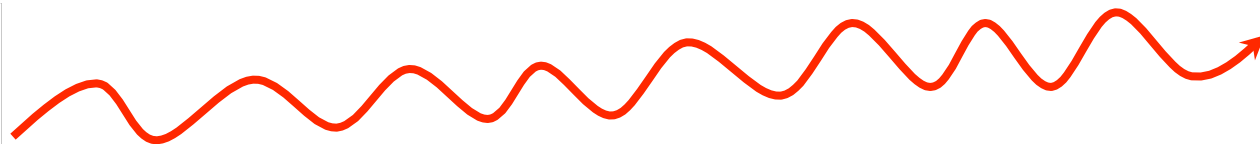
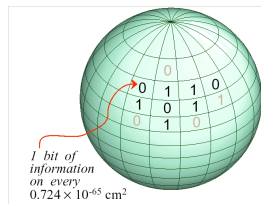


Holography 1: Black Hole Thermodynamics

- Beckenstein, Bardeen et al. (~1972): laws of black hole thermodynamics
- Area of event horizon, like entropy, always increases
- Entropy is identified with $1/4$ of event horizon **area** in Planck units (not volume)
- Is there is a deep reason connected with microscopic degrees of freedom of spacetime encoded on the surface?

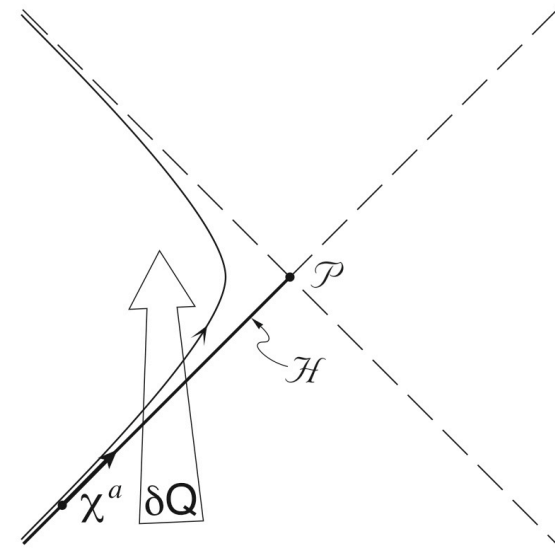
Holography 2: Black Hole Evaporation

- Hawking (1975): black holes radiate ~thermal radiation, lose energy and disappear
- Is information lost? Or is quantum unitarity preserved?
- Degrees of freedom: evaporated quanta carry degrees of freedom (~ 1 per particle) as area decreases
- Black hole entropy may completely account for information of evaporated states, also assembly histories
- Is black hole completely described by information on 2+1D event horizon?
- Information of evaporated particles = entropy of hole



Holography 3: nearly-flat spacetime

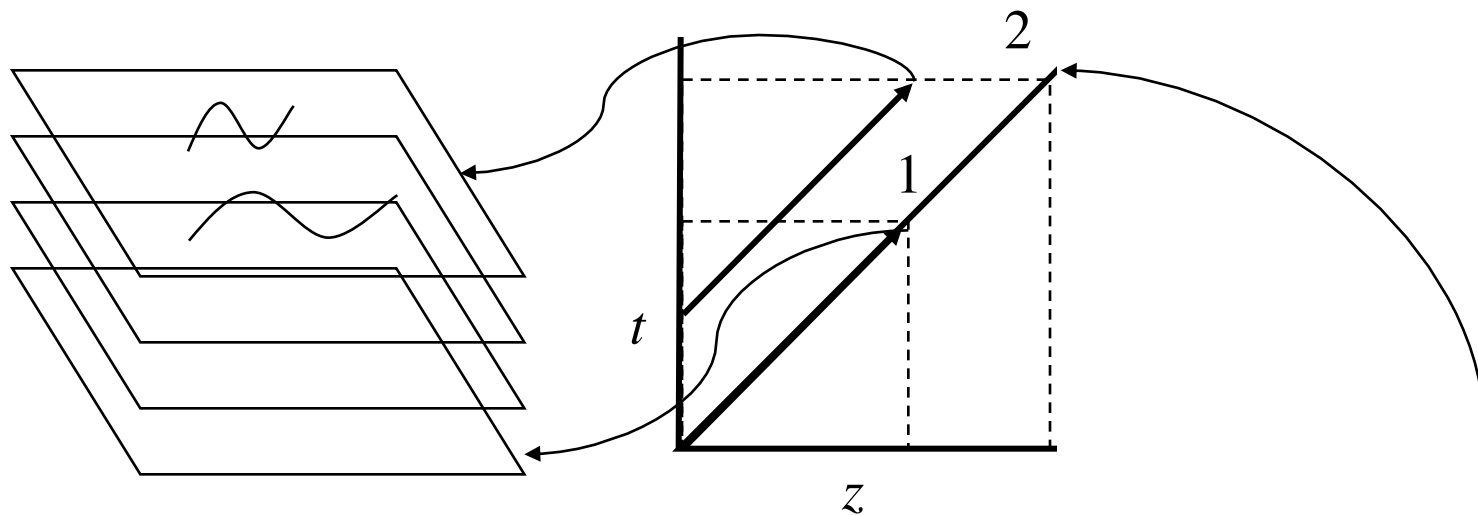
- Unruh (1976): Hawking radiation seen by accelerating observer
- Appears with any event horizon, not just black holes: identify entropy of thermal radiation with missing information
- Jacobson (1995): Einstein equation derived from thermodynamics (\sim equation of state)
- Classical GR from 2+1D null surface (Padmanabhan 2007)



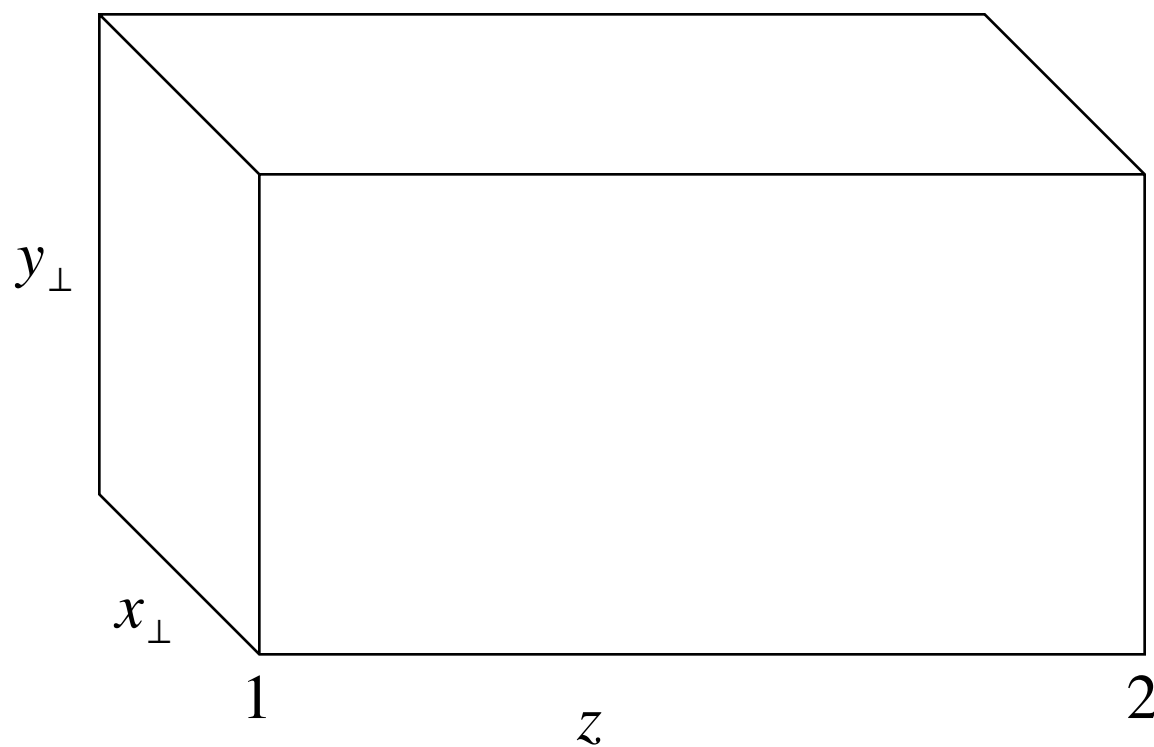
Jacobson: points=2D surfaces

Holography 4: Covariant (Holographic) Entropy Bounds

- 't Hooft (1985): black holes are quantum systems
- 't Hooft, Susskind et al. (~1993): world is "holographic", encoded in 2+1D at the Planck scale
- Black hole is highest entropy state (per volume) and sets bound on entropy of any system (includes quantum degrees of freedom of spacetime)
- All physics within a 3D volume can be encoded on a 2D bounding surface ("holographic principle")
- Bousso (2002): holographic principle generalized to "covariant entropy bound" based on causal diamonds: entropy of 3D light sheets bounded by area of 2D bounding surface in Planck units
- Suggests that 3+1D geometry emerges from a quantum theory in 2+1D: light sheets



*3+1D spacetime
emerging from
2+1D: light
sheet with $z=t$*



Holography 5: hints from string/M theory

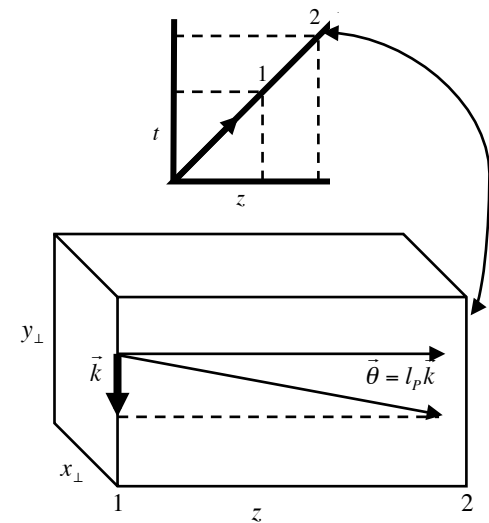
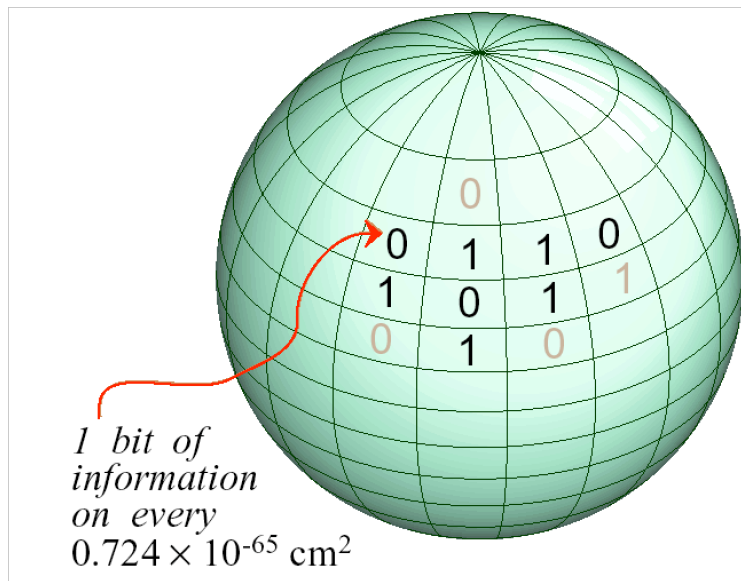
- Strominger, Vafa (1996): count degrees of freedom of extremal higher-dimension black holes using duality
- All degrees of freedom appear accounted for
- Agrees with Hawking/Beckenstein thermodynamic count
- Unitary quantum system (but zero temperature)
- Strong indication of a minimum length \sim Planck length
- What do the degrees of freedom look like in a realistic system?
- Matrix theory (Banks et al. 1996): Hamiltonian with noncommuting transverse position observables agrees with string duals, holographic scaling of states
- Hints at uncertainty principle for transverse position
- Compactification scales: below Planck, extra dimensions act like particle degrees of freedom

Holography 6: Exact dual theories in $N-1$ dimensions

- Maldacena, Witten et al. (1997...): AdS/CFT correspondence
- N dimensional conformal field "boundary" theory exactly maps onto (is dual to) $N+1$ dimensional "bulk" theory with gravity and supersymmetric field theory
- Alishahiha et al.: de Sitter spacetime in N dimensions maps onto de Sitter in $N+1$, $N-1$
- Is nearly flat $3+1$ spacetime described as a dual in $2+1$?

Holographic quantum geometry implements covariant (holographic) entropy bound in emergent 3+1D spacetime

- Reflects Hilbert space of 2+1D theory
- By construction, follows light sheets of covariant Bousso formulation
- Far fewer independent modes than field theory quantized in 3+1D
- independent pixels in 3D volume \sim area of 2D null surface element



A holographic world is blurry

limited information content



What does it look like
"from inside"?
("Flatland" realized with
waves)



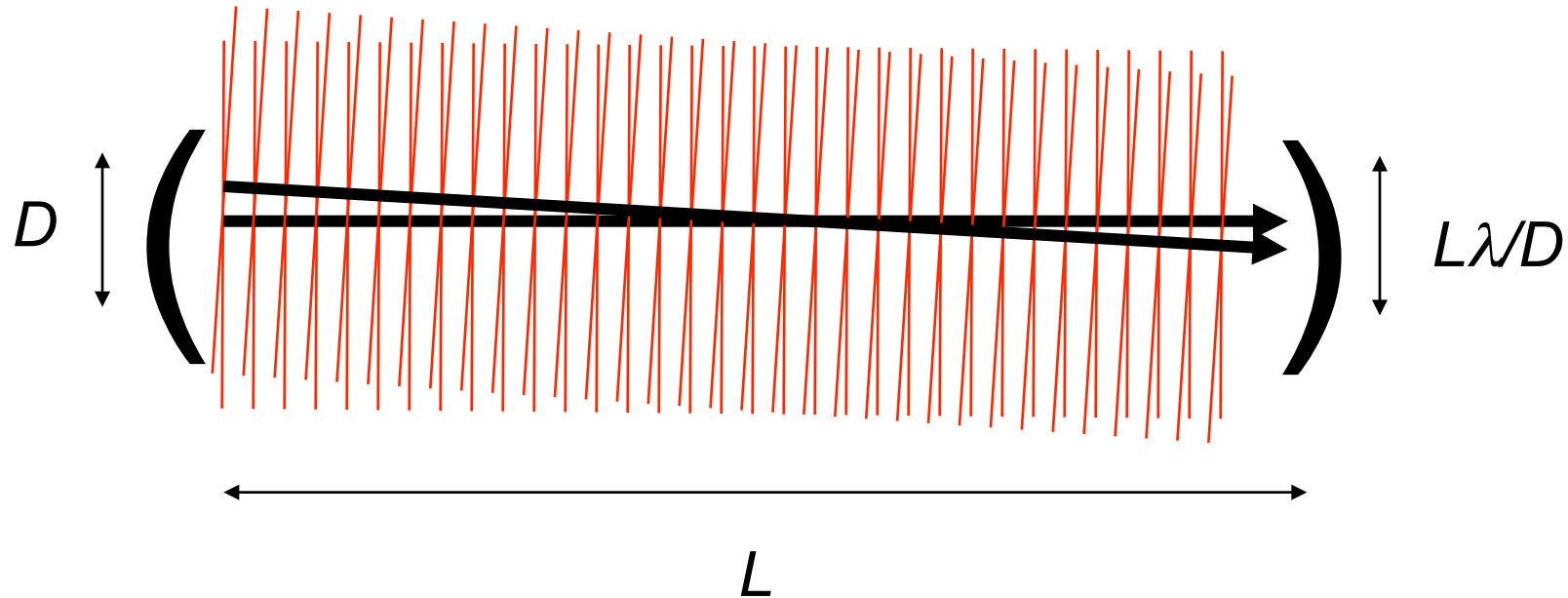
Holographic Quantum Geometry

- Spacetime is a quantum system
- **Conjecture: the world is formed by Planck wavelength null waves**
- "from inside": transverse quantum fluctuations in position much larger than Planck length

$$l_P = \sqrt{\hbar G_N / c^3} = 1.616 \times 10^{-33} \text{cm}$$



Ray limit of wave optics: Rayleigh uncertainty



- Aperture D , wavelength λ : angular resolution λ/D
- Size of diffraction spot at distance L : $L\lambda/D$
- Endpoints of a ray can be anywhere in aperture, spot
- path is determined imprecisely by waves
- Minimum uncertainty at given L when aperture size = spot size, or

$$D = \sqrt{\lambda L}$$

The case of a real hologram

- For optical light and a distance of about a meter,

$$D = \sqrt{\lambda L}$$

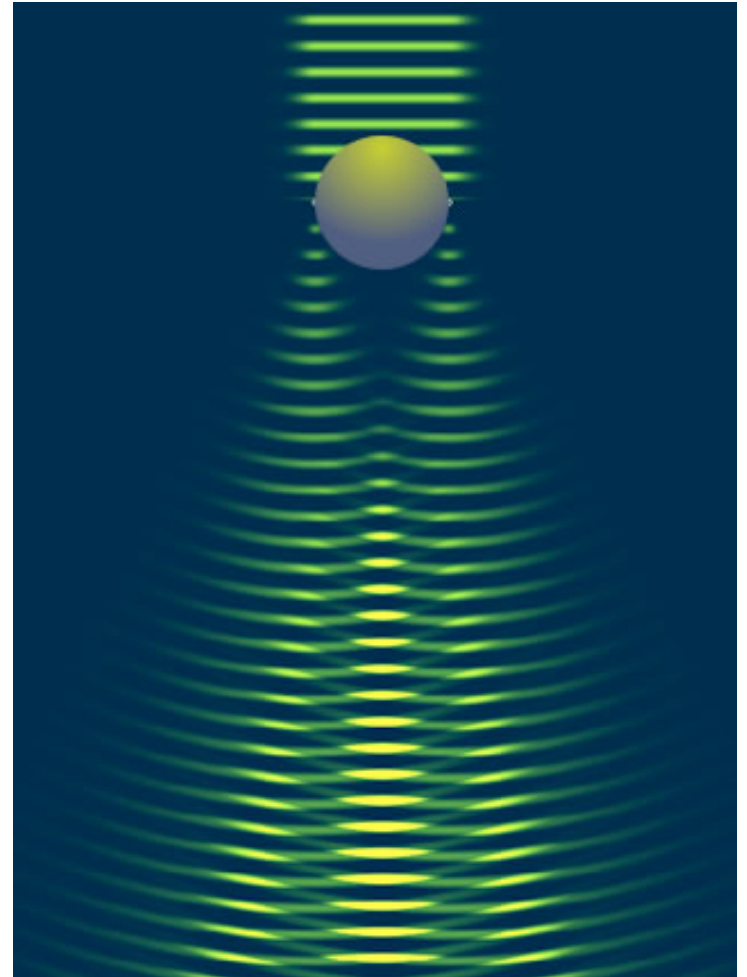
is about a millimeter

- Larger aperture gives sharper image but then photon paths and arrival positions cannot be measured so well
- If you "lived inside" a hologram, you could tell by measuring the blurring/indeterminacy



Wave Theory of Spacetime Indeterminacy

- Adapt theory of transverse correlation in wave optics
- theory of “position wavefunctions”
- Complex amplitude=wavefunction
- Complex correlation=quantum correlation
- Intensity=probability
- Set wavelength to match holographic degrees of freedom



Uncertainty of transverse position

Spacetime positions are wavefunctions. Transverse positions at separation L events are Fourier transforms of each other and have standard deviations related by:

$$\sigma' \sigma = \lambda L$$

For macroscopic L the “uncertainty” is much larger than the wavelength

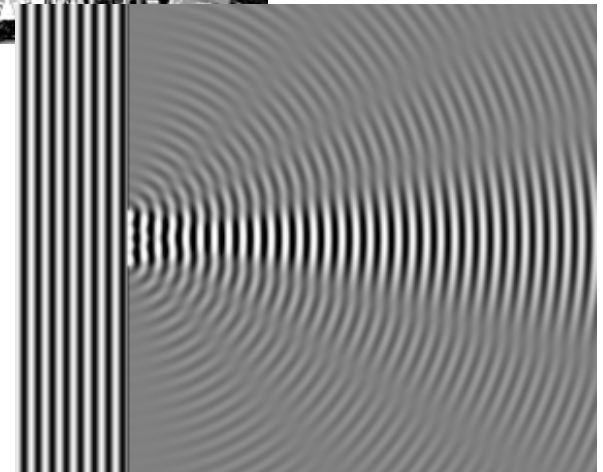
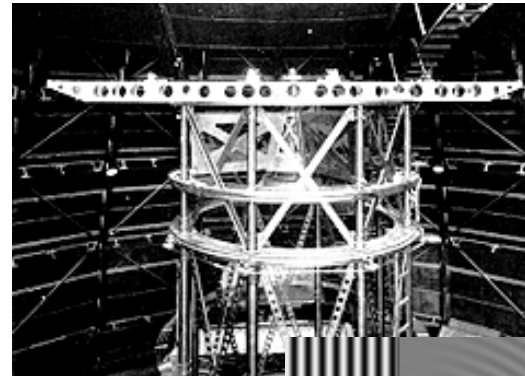
Controlled covariant theory based on wave
optics: CJH, arXiv: **0806.0665**

Familiar examples from the world of optics

- Hanbury Brown-Twiss interferometry: correlation of intensity from distant star in widely separated apertures
- Michelson stellar interferometer: fringes from star
- Diffraction in the lab: shadow of plane wave cast by edge or aperture

All display similar optical examples of wave phenomena much larger than the waves,

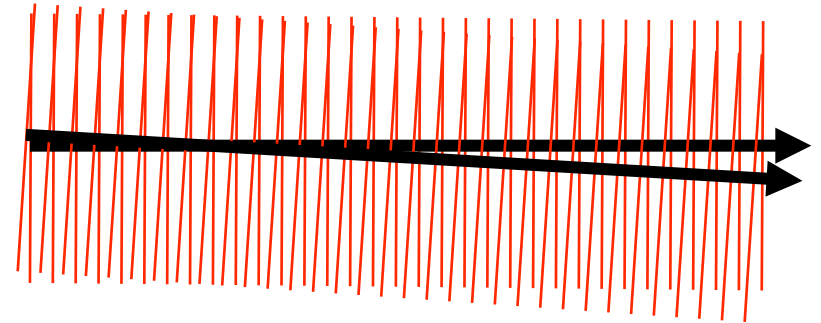
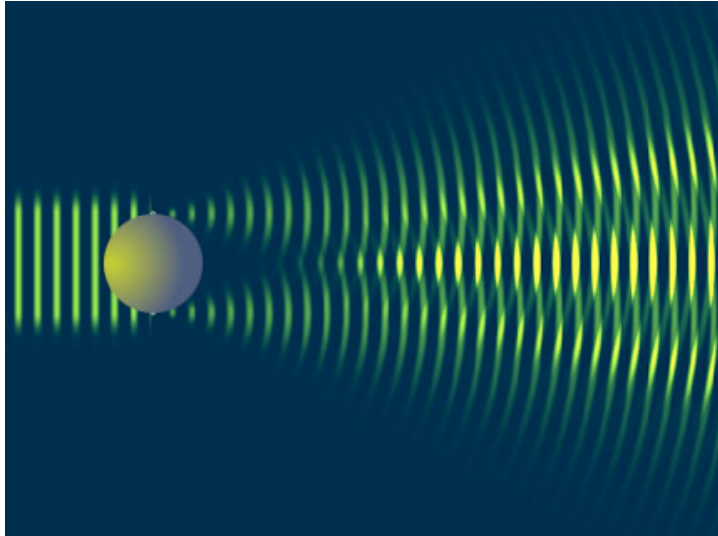
$$\sigma' \sigma = \lambda L$$



Set fundamental wavelength= Planck length

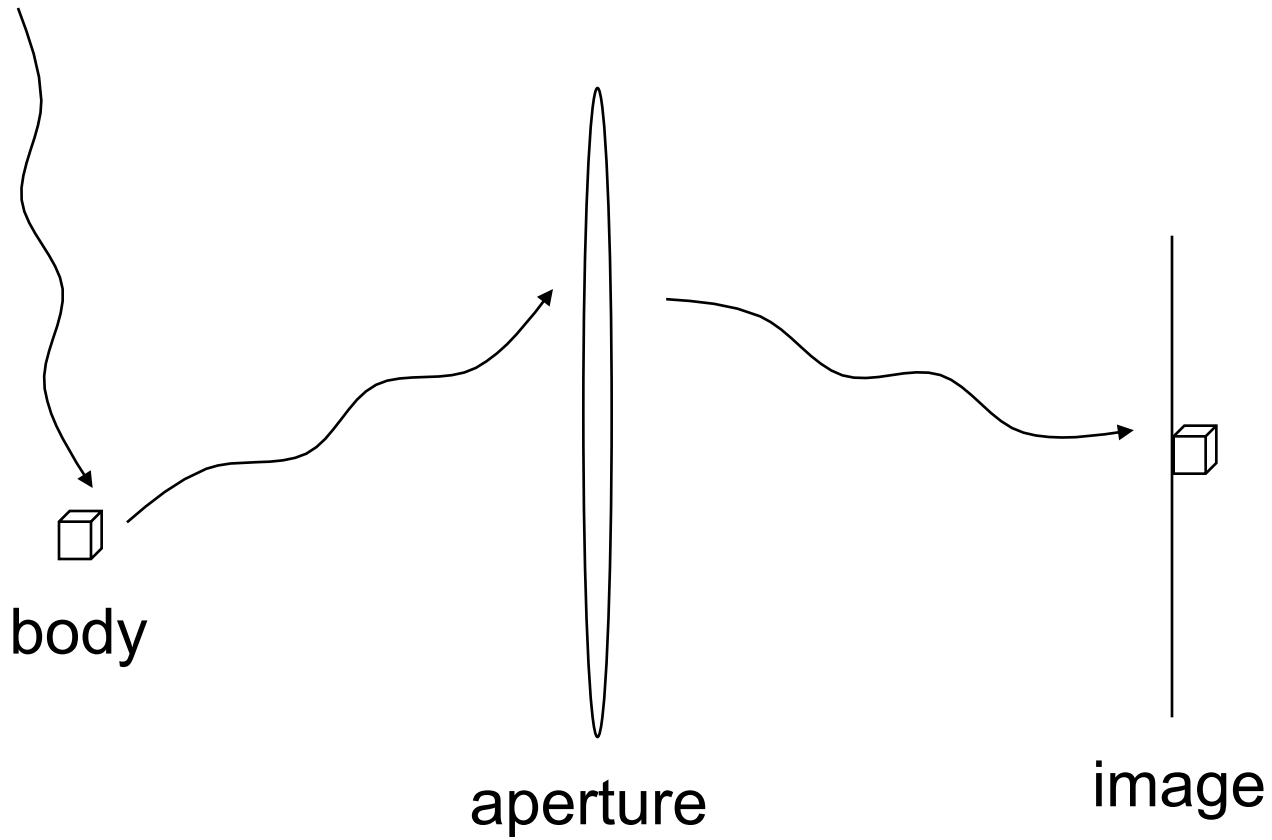
- Agrees with degrees of freedom, minimum wavelength from gravitation theory
- black hole evaporation, unitary behavior of gravitational systems
- Sets absolute normalization with no parameters
- No gravity: flat wavefronts, flat space

Indeterminacy of a Planckian path: diffraction limit of "Planck wavelength telescope"



- spacetime metric defined by paths between events
- Complementarity: path \sim ray approximation to waves
- Transverse wavefunction of events displays indeterminacy formally identical to optical wave correlations
- Indeterminacy of geometry reflects limited information content of waves

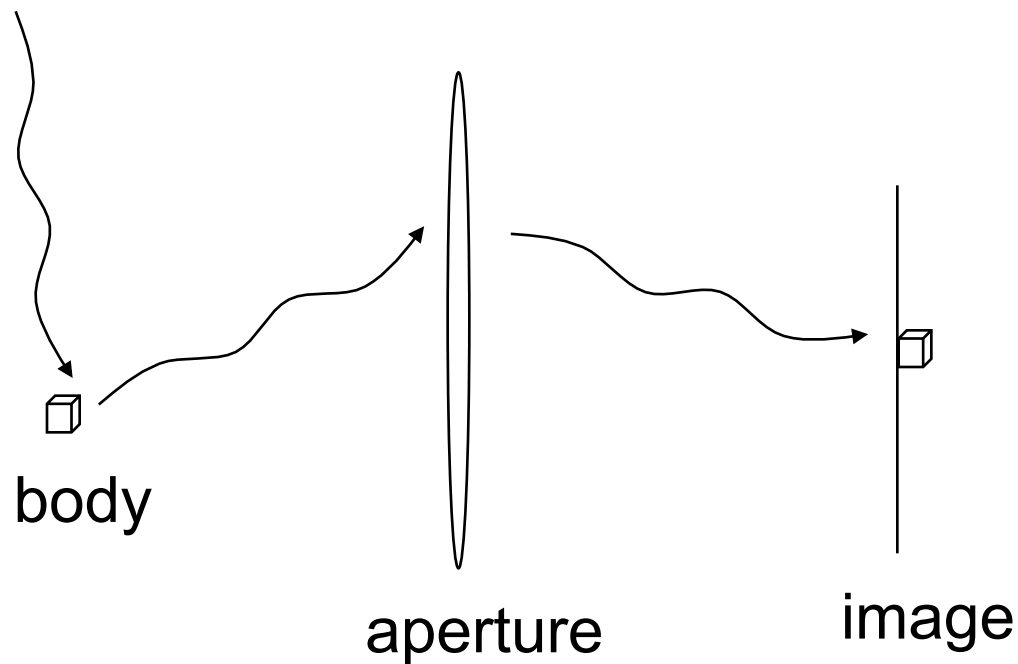
"Heisenberg microscope"



$$\Delta(\text{measured position}) \times \Delta(\text{momentum of perturbation}) > \hbar/2$$

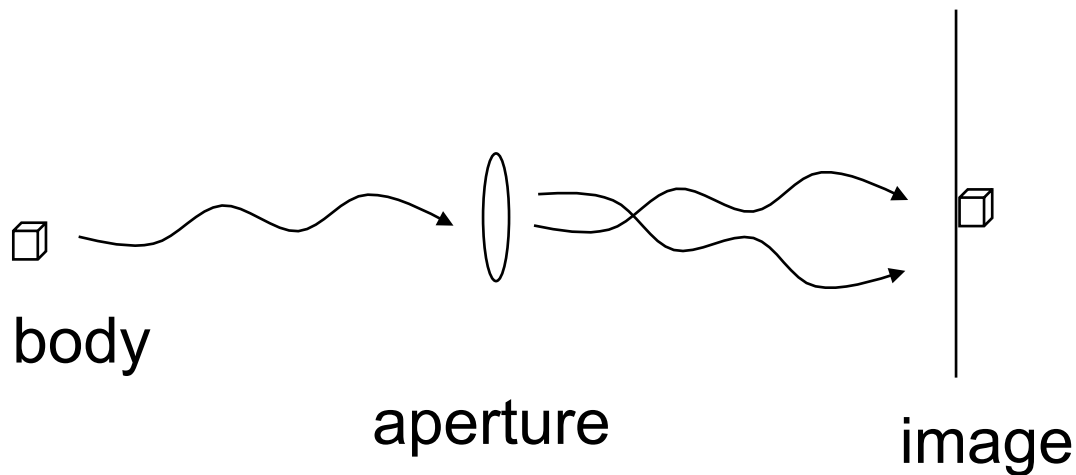
Heisenberg Microscope

- Measures transverse position by imaging using scattered light
- Complementarity between measured position, transverse photon momentum
- observables do not have independent classical meaning



"Planck telescope"

- Create image on a screen with pinhole aperture
- minimum wavelength= Planck
- Minimum uncertainty in angle or transverse position when size of pinhole~ size of its own diffraction spot
- Transverse positions do not have independent classical meaning



Uncertainty: Heisenberg and Holographic

- "Heisenberg microscope":
transverse position of a remote body measured by angular position ~ detected position of radiation particle in image
- Fixed 3D classical space
- $\Delta(\text{measured transverse position of a body}) \times \Delta(\text{momentum of measuring radiation}) > \hbar/2$
- Δ independent of microscope aperture, focal length
- Property of body, radiation
- State of body, radiation depends on measurement
- "Planck telescope": transverse position of remote events measured by Planck radiation
- Position observables: 2D apertures with Planck waves
- $\Delta(\text{position 1}) \times \Delta(\text{position 2}) > (\text{Planck length}) \times (\text{separation})$
- Δ position ~ optimal "aperture", depends on separation
- Property of (quantum) spacetime geometry: **limiting precision of Planck imaging**
- **State of metric depends on measurement**

The background image is a detail from Plato's Academy by Raphael. It depicts Plato pointing upwards towards the sky, while Aristotle below him gestures towards the earth. They are surrounded by other figures in classical attire, some engaged in discussion and others in study. The setting is a grand architectural space with arches and statues.

geometry is not classical: it depends on measurement

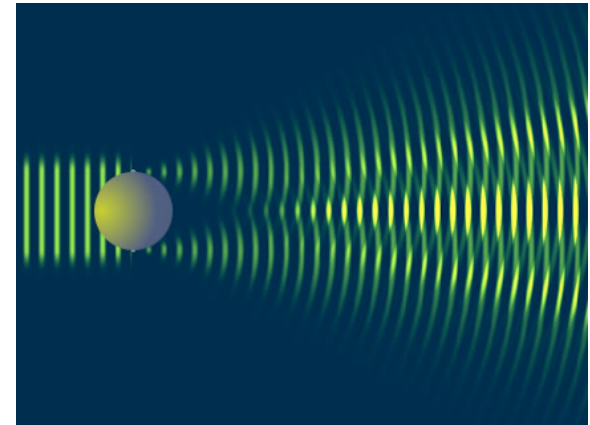
holographic approach to the classical limit

- **Angles** are indeterminate at the Planck scale, and become better defined at larger separations:

$$\Delta\theta(L) = (l_P/L)^{1/2}$$

- But uncertainty in **relative transverse position increases** at larger separations:

$$\Delta x_{\perp}^2 > l_P L$$

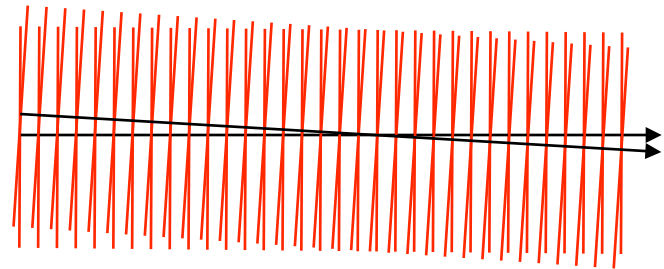
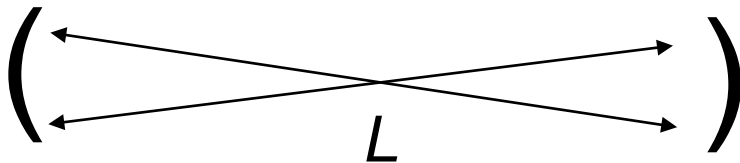


- Not the classical limit of field theory
- Indeterminacy and nonlocality persist to macroscopic scales

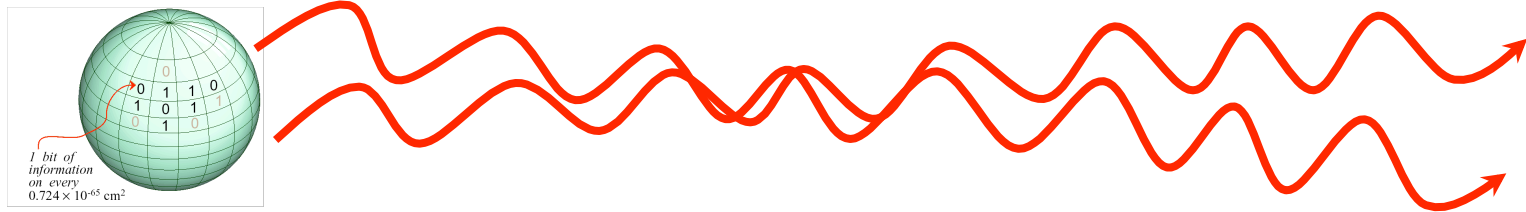
Holographic indeterminacy in angle

Angular orientation of a null path defined by Planck wavelength particles is uncertain, with standard deviation:

$$\Delta\theta(L) = (l_P/L)^{1/2}$$



Holographic indeterminacy of distant spacetime allows black hole evaporation to be a reversible unitary quantum process



- ~one degree of freedom leaves the hole for each evaporated particle, wavelengths of order hole size
- If the quantum states of the evaporated particles allowed transverse position observables with 3D Planck precision, at large distance they would contain more information than the hole
- With holographic indeterminacy of distant spacetime, distant positions are not all distinguishable states and the problem disappears

Fluctuations in quantum geometry

- Distant positions only defined as well as Planck waves
- Widely separated positions have fuzzy quantum relationship
- Indeterminacy leads to fluctuations in measured quantities
- **Transverse positions are noncommuting observables**
- Statistical predictions not sensitive to model details: direct measure of fundamental information bounds

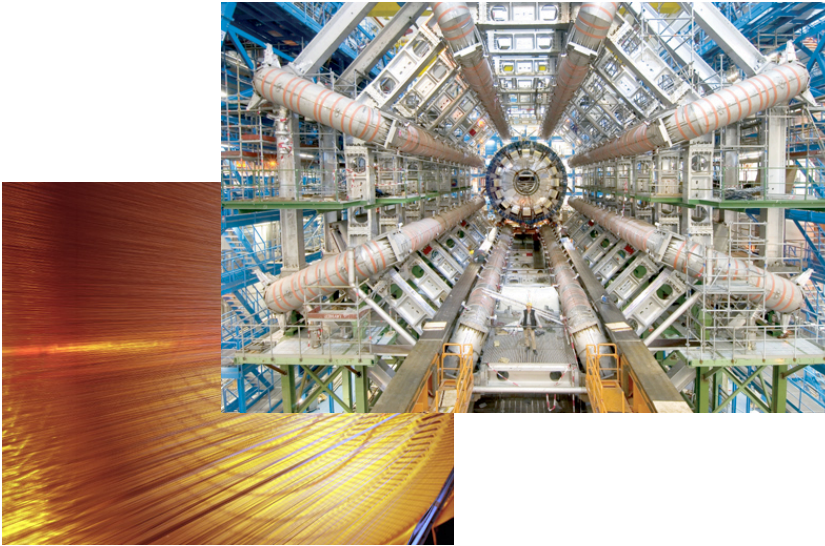


Interferometers as Planck telescopes

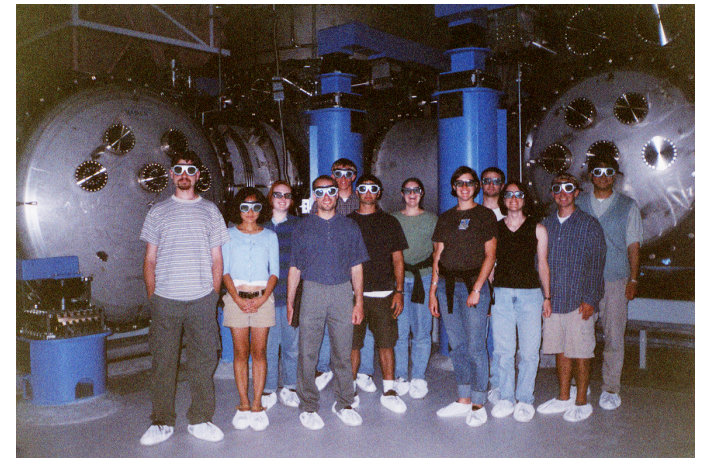
- Nonlocality: precise relative positions at km scales
- Fractional precision: angle $< 10^{-20}$, > "halfway to Planck"
- Transverse position measured in some configurations
- Precision: like two collisions at LHC localized at exactly the same place after a complete circuit
- Heavy proof masses, small Heisenberg uncertainty: positions measure spacetime wavefunction
- Detect holographic blurring from noise in signal

microscope for measuring quantum geometry requires
coherent quantum state over macroscopic distance

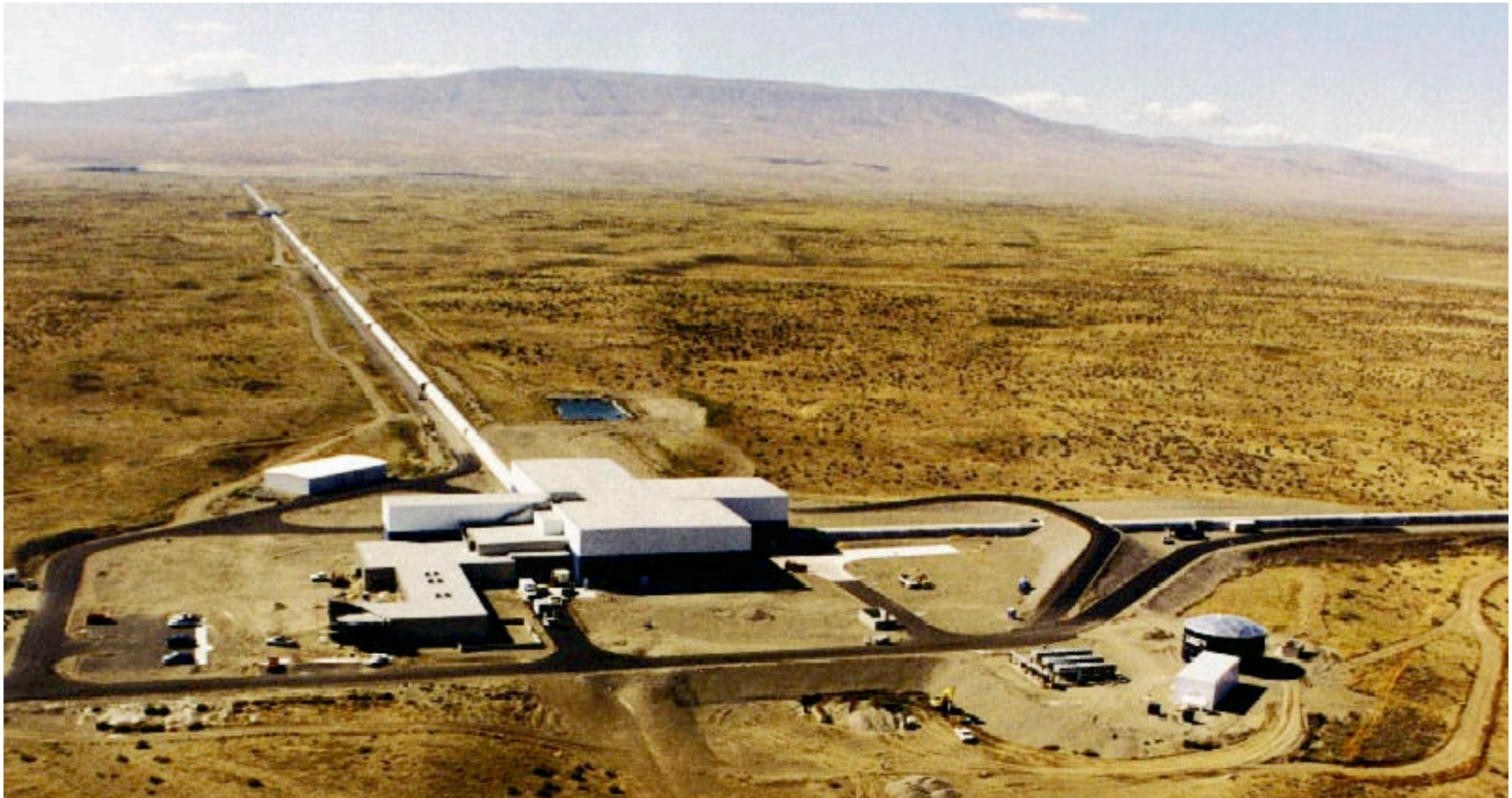
CERN/FNAL: $\text{TeV}^{-1} \sim 10^{-18} \text{ m}$



LIGO/GEO: $\sim 10^{-19} \text{ m}$
over $\sim 10^3 \text{ m}$ baseline

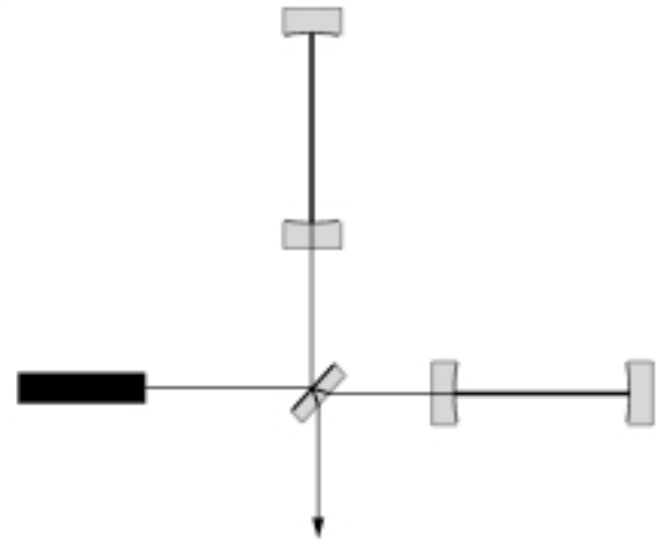
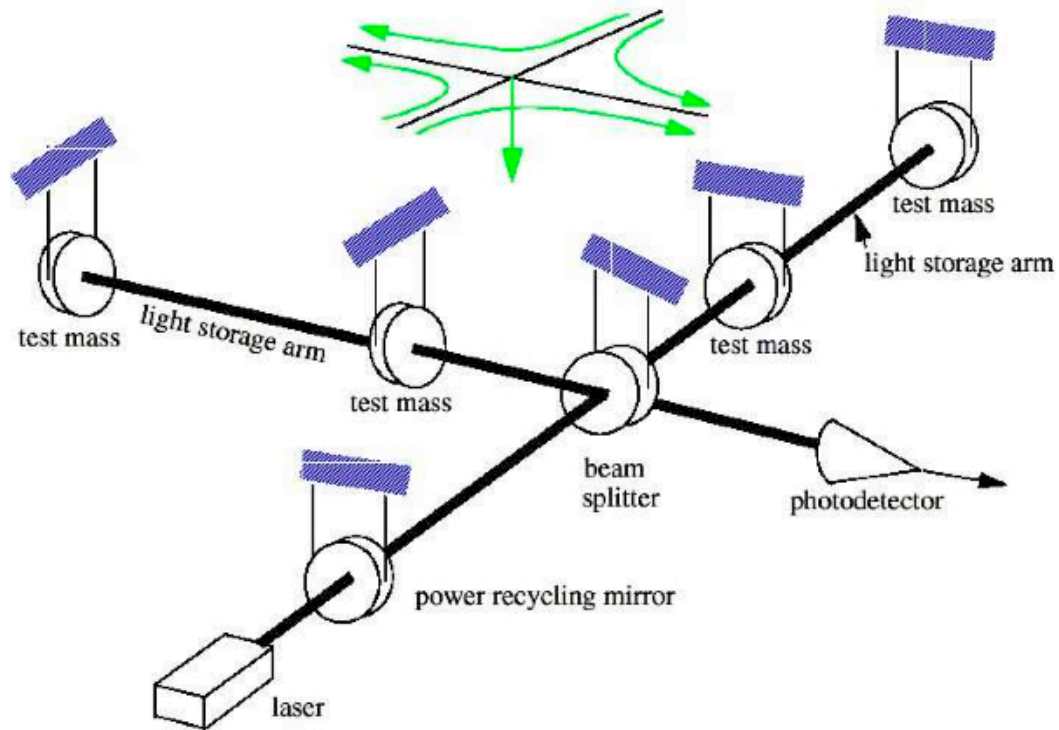


LIGO Hanford Observatory

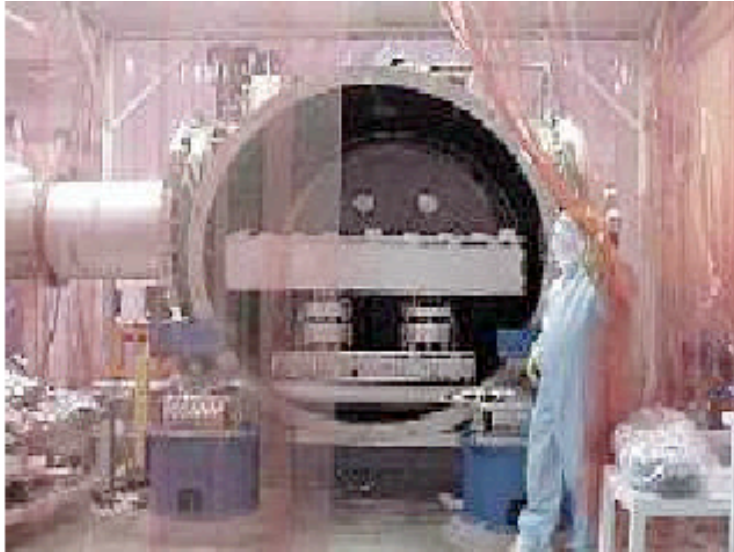


Schematic layout of LIGO

Fig. 1. Schematic layout of a LIGO interferometer.







vibration-isolated platform

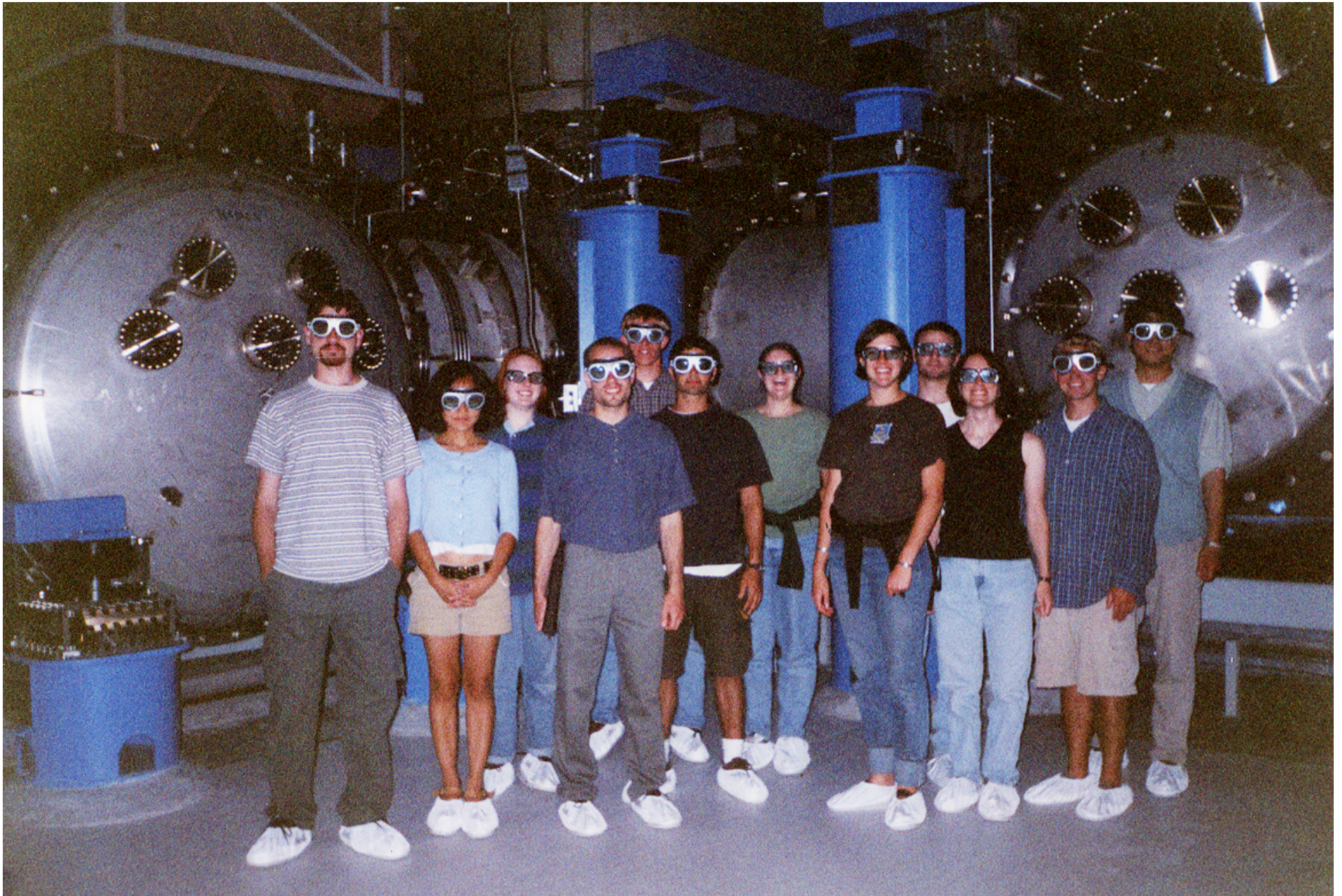


initial alignment

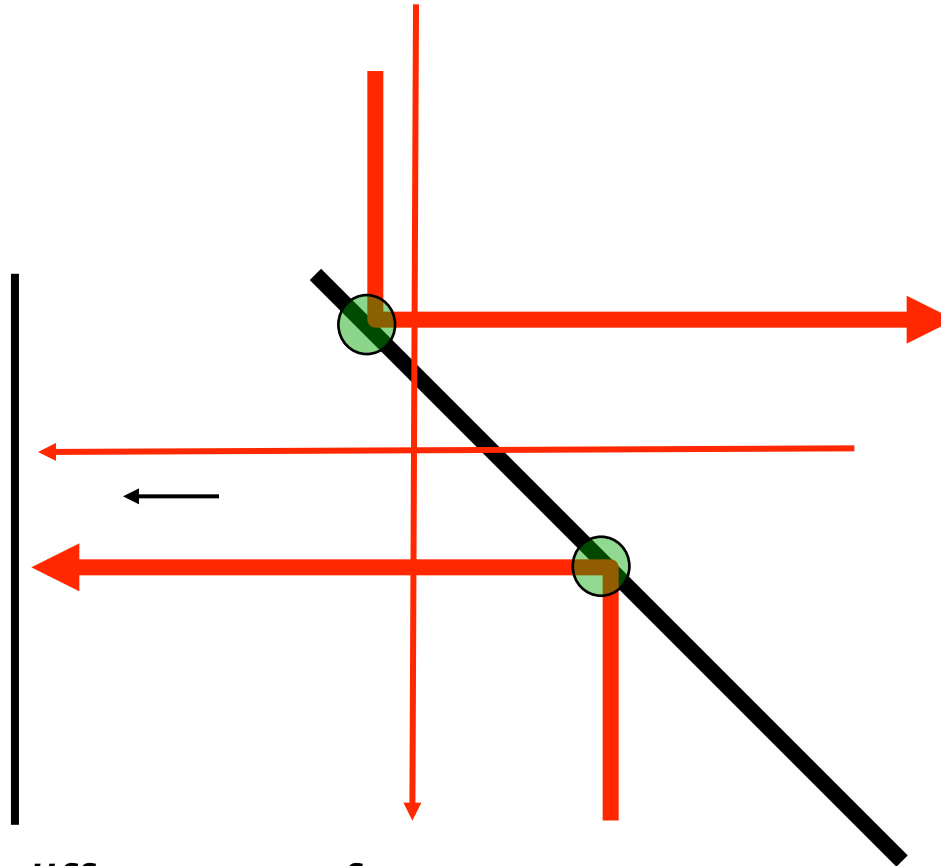


test mass suspended on fine wire

UW Physics undergrads at LIGO Hanford Observatory



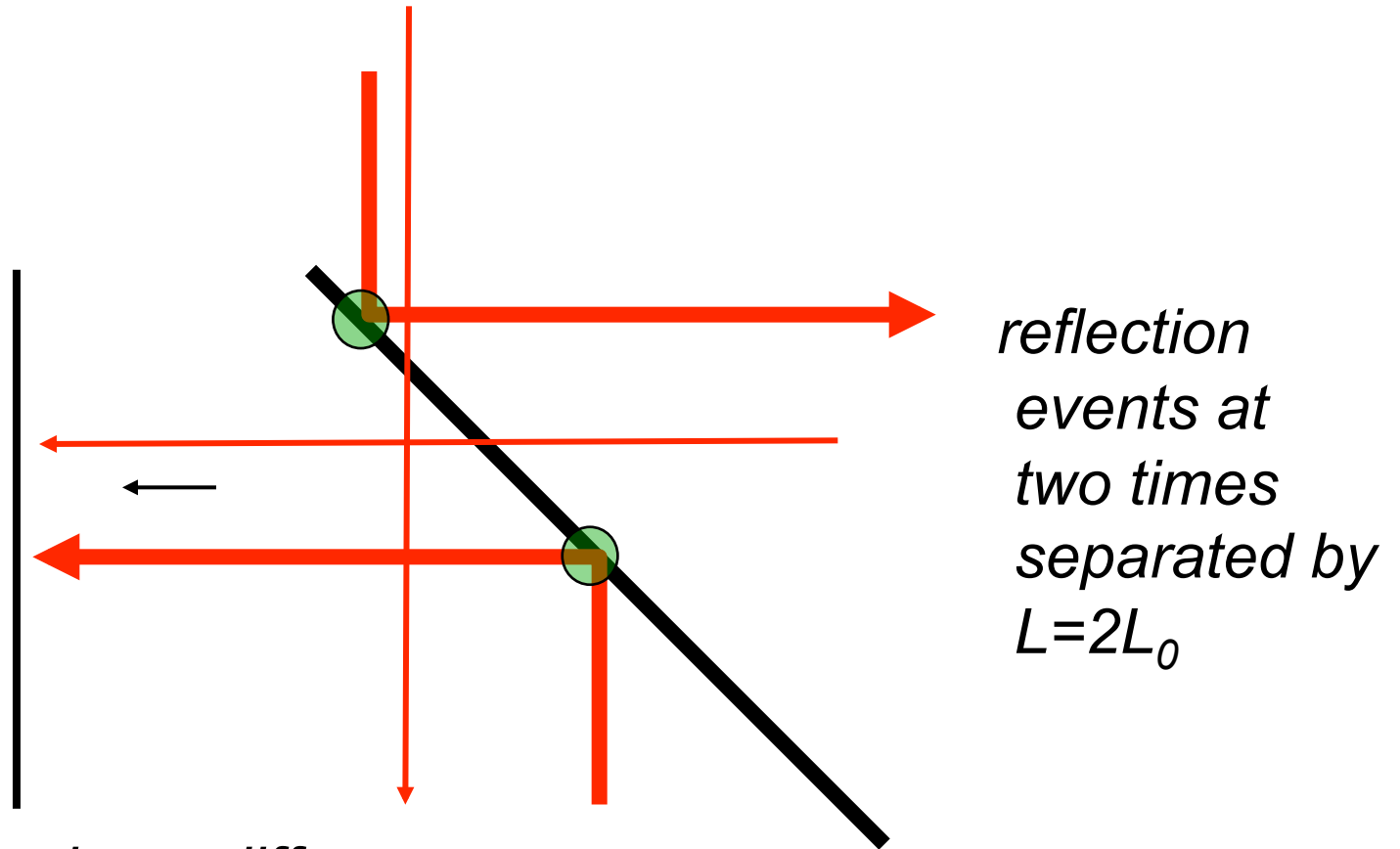
Beamsplitter and signal in Michelson interferometer



*Signal phase ~ difference of
integrated distance along two
orthogonal arms*

Beamsplitter

Holographic noise in the signal of a Michelson interferometer



Signal: random phase difference of reflection events from indeterminate position difference of beamsplitter at the two events

Quantum uncertainty of transverse positions of beamsplitter

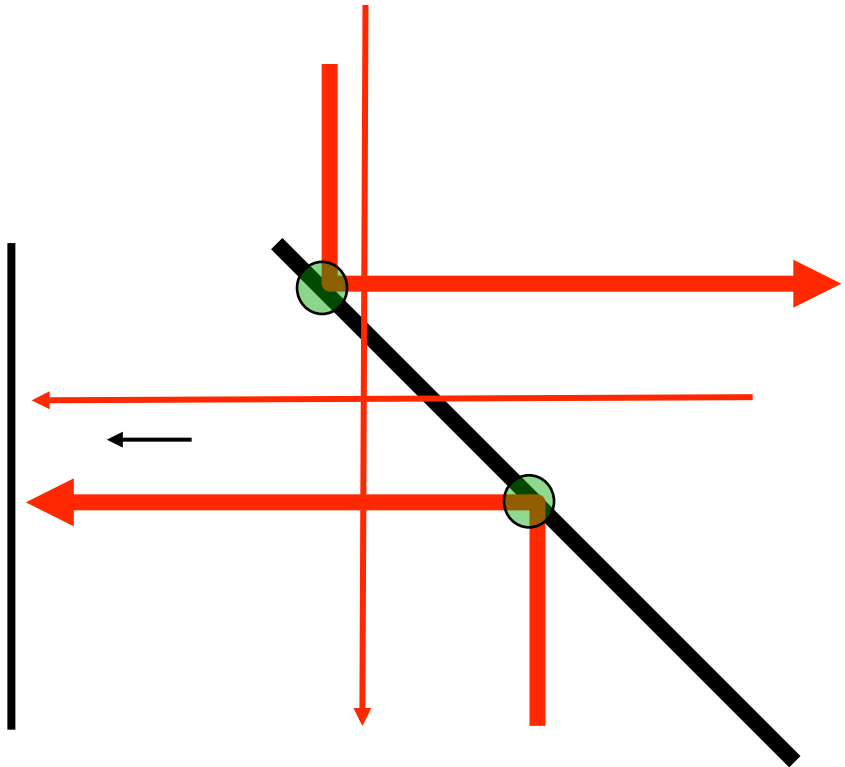
- Positions of reflection events have transverse uncertainties

$$\sigma' \sigma = \lambda L$$

- Independent samplings accumulate signal phase uncertainty
- apparent arm length difference is a random variable, with variance:

$$\Delta L_0^2 = \sigma^2 + \sigma'^2 = 2\sigma^2 = 4l_P L_0$$

this is a new effect predicted with no parameters



Power Spectral Density of Fluctuations

Uncertainty in angle \sim dimensionless shear

$$\Delta\theta(L) = (l_P/L)^{1/2}$$

\sim shear fluctuations with flat power spectral density

$$h_H^2 \simeq L\Delta\theta^2 \approx t_P$$

h_H^2 = mean square perturbation per frequency interval

(prediction with no parameters, Planck length is the only scale)

Holographic Noise

*Universal **holographic noise** ~ flat power spectral density of **shear** perturbations:*

$$h \approx \sqrt{t_P} = 2.3 \times 10^{-22} \text{Hz}^{-1/2}$$

- general property of holographic quantum geometry
- Prediction of spectrum with no parameters
- Prediction of spatial shear character: only detectable in transverse position observables
- Definitively falsifiable
- Contrast with more general range of possible phenomenology (e.g. Amelino-Camelia, et al.)

Holographic fluctuations do not carry energy or information

- ~ classical gauge mode (flat space, no classical spacetime degrees of freedom excited)
- ~ sampling noise, not thermal noise
- Necessary so the number of distinguishable positions does not exceed holographic bound on Hilbert space dimension
- No curvature
- no strain, just shear
- no detectable effect in a purely radial measurement

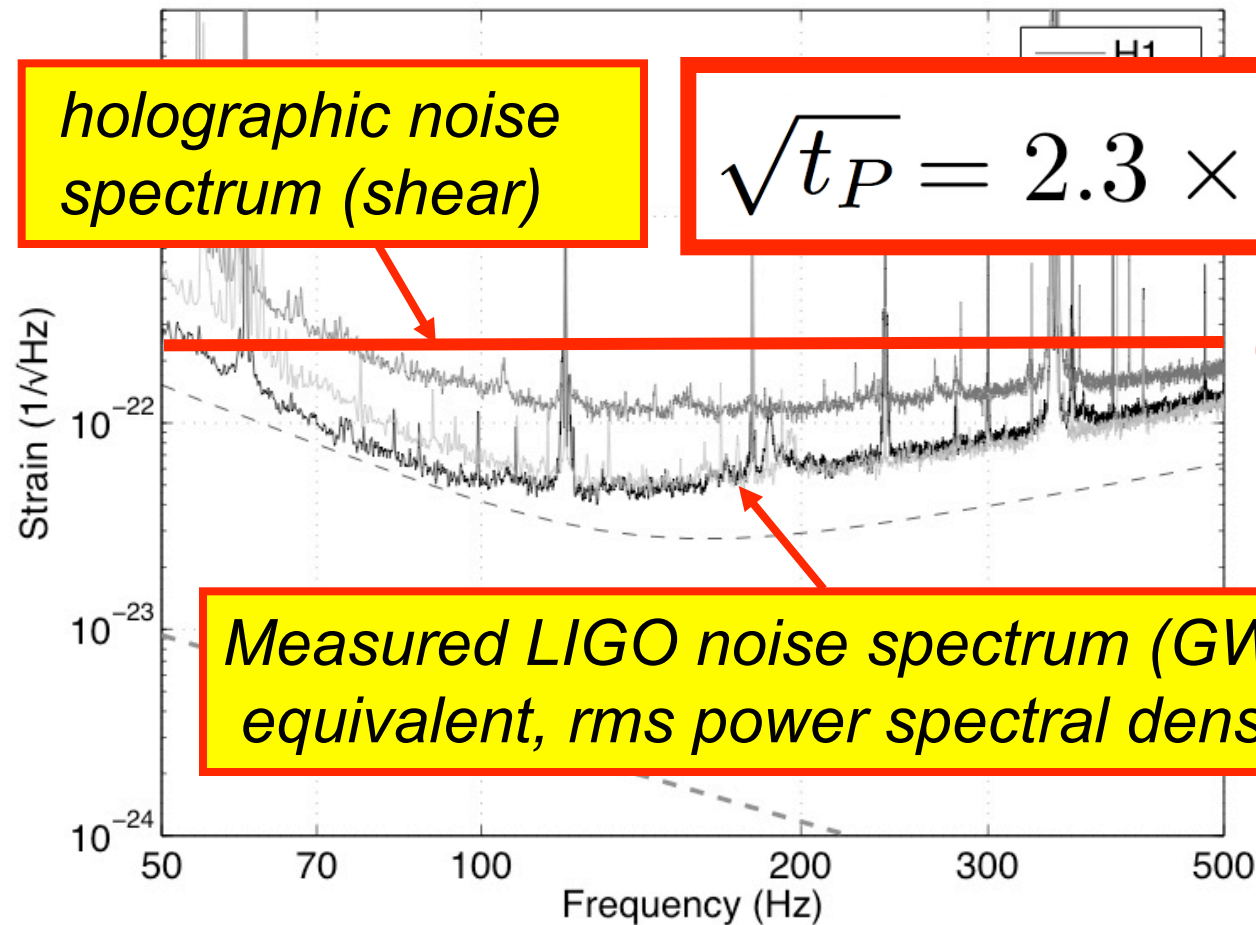
LIGO noise (astro-ph/0608606)



*holographic noise
spectrum (shear)*

$$\sqrt{t_P} = 2.3 \times 10^{-22} / \sqrt{\text{Hz}}$$

(if shear=strain)

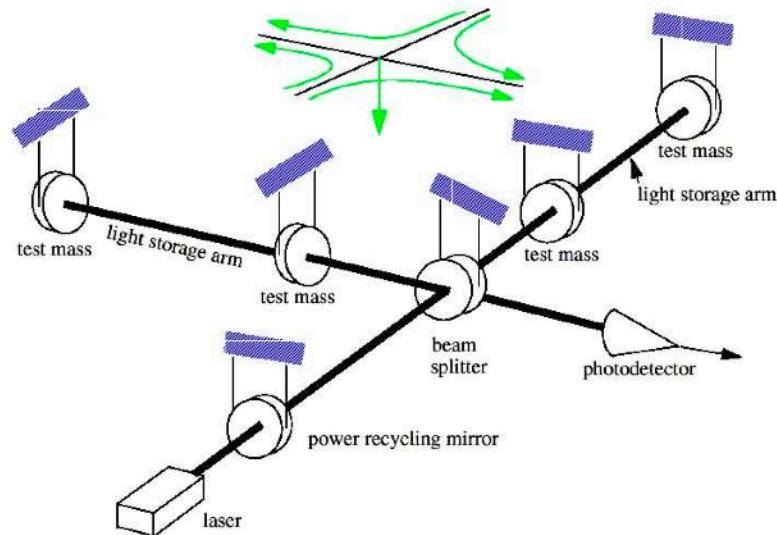


*Measured LIGO noise spectrum (GW strain
equivalent, rms power spectral density)*

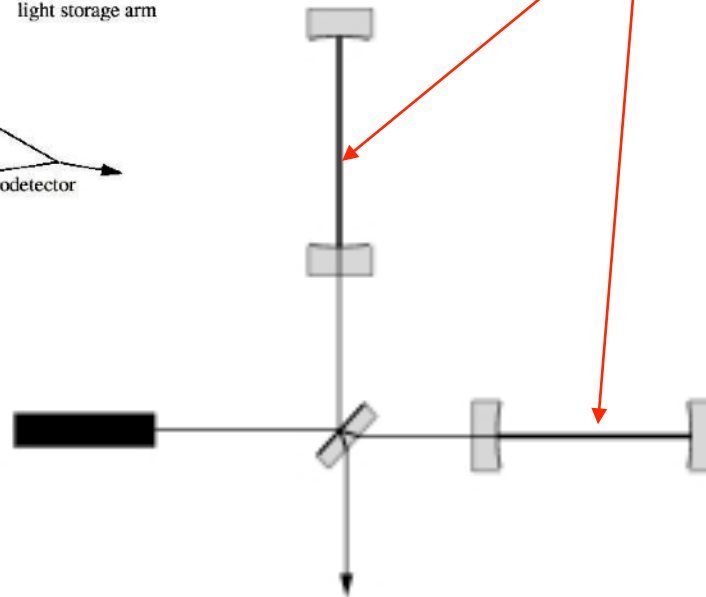
Why doesn't LIGO detect holographic noise?

- EITHER holographic noise does not exist, OR:
- LIGO layout is not sensitive to transverse displacement noise (relationship of holographic to gravitational wave depends on details of the system layout)

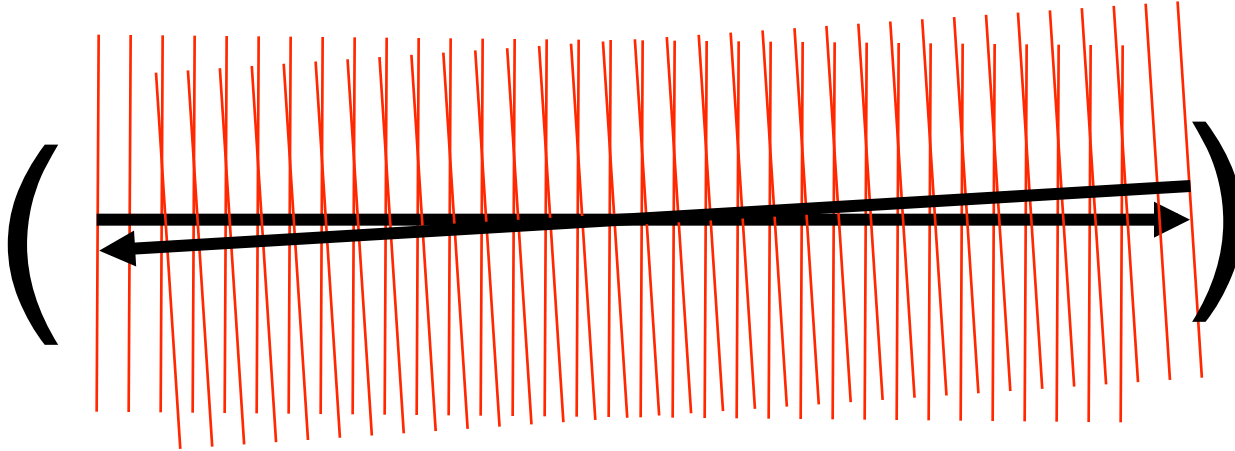
Fig. 1. Schematic layout of a LIGO interferometer.



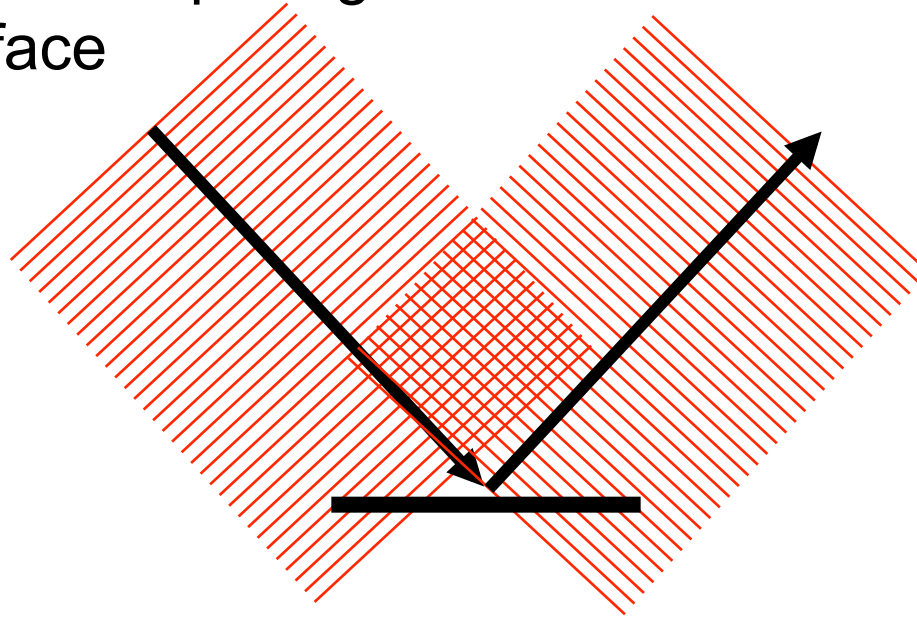
Transverse position measurement is not made in FP cavities



Normal incidence optics: phase signal does not record the transverse position of a surface

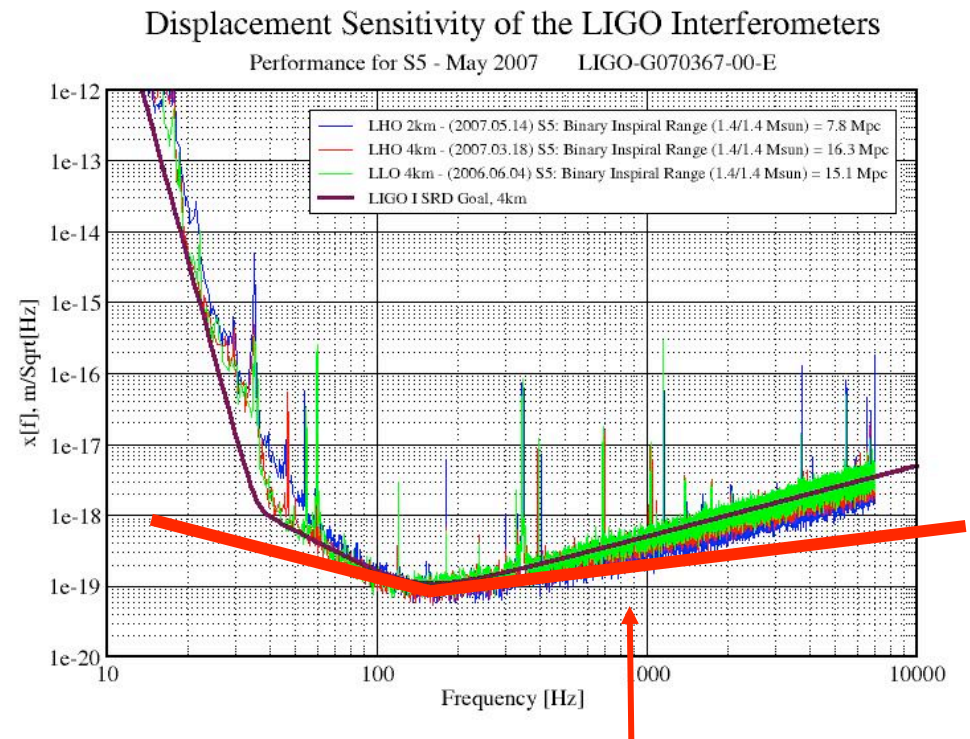


- But phase of beam-split signal is sensitive to transverse position of surface



LIGO S5 run: noise in displacement units

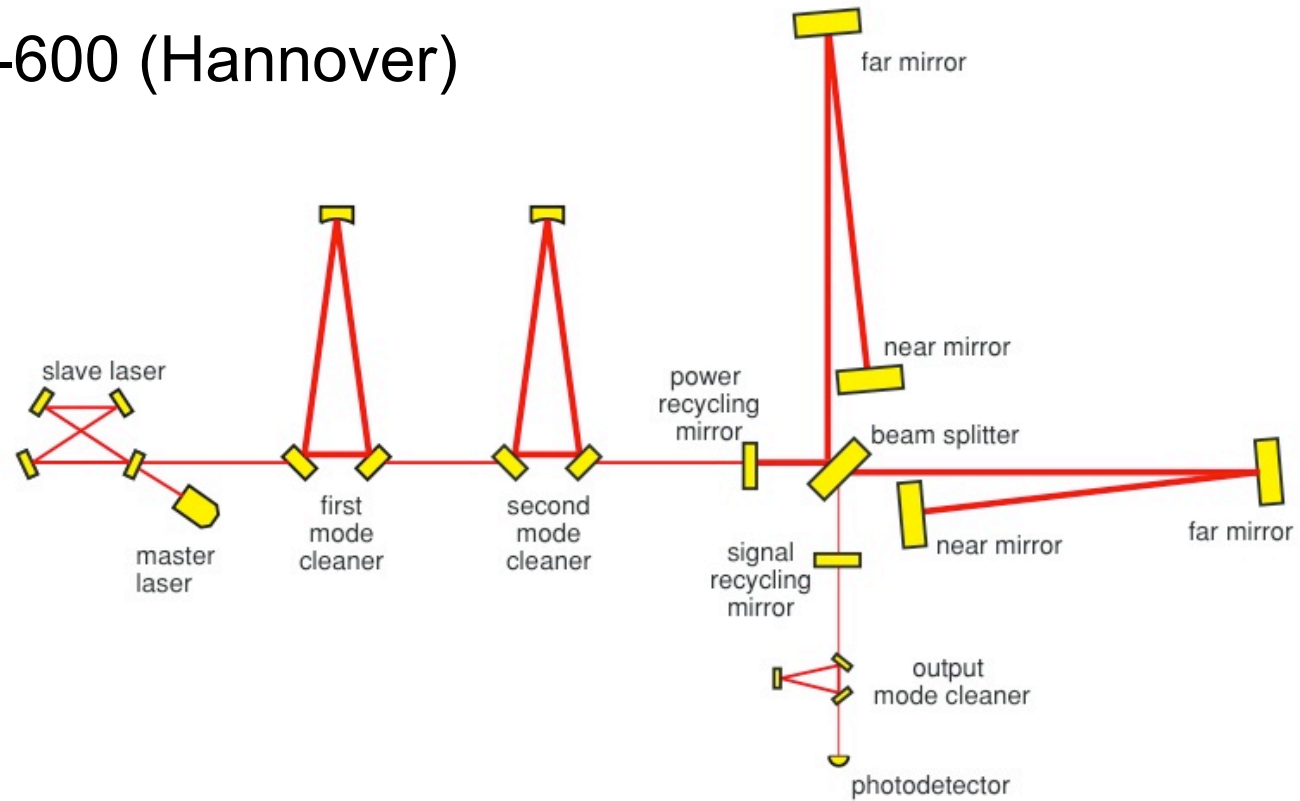
- Allow for lack of holographic noise from FP arm cavities
- In displacement units, estimated holographic noise is below sensitivity of last science run
- Will be detectable with enhanced/advanced LIGO



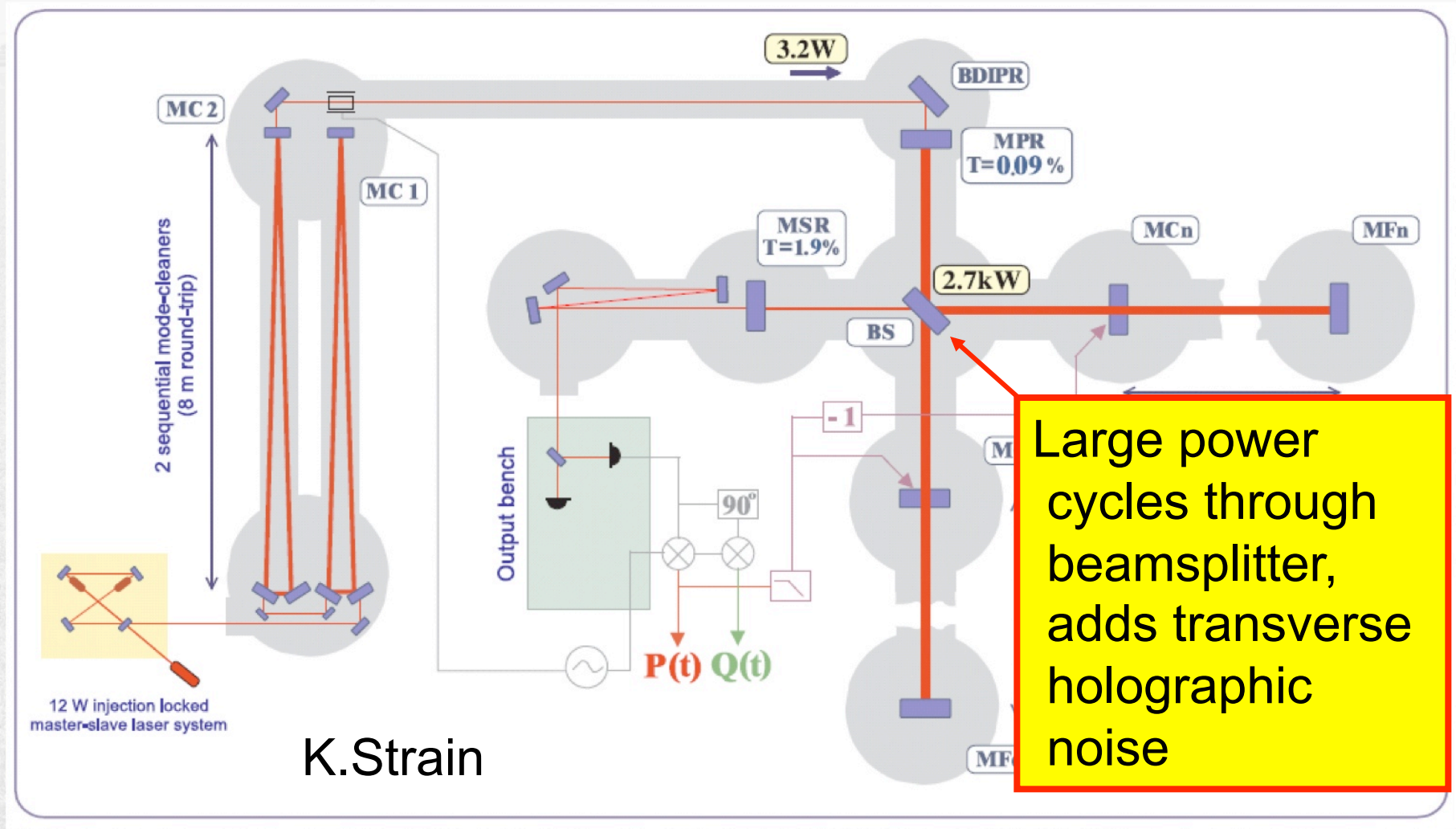
Rough but zero-parameter estimate of holographic noise in LIGO (displacement units)

CJH: [arXiv:0806.0665](https://arxiv.org/abs/0806.0665)

GEO-600 (Hannover)



The GEO600 Interferometer

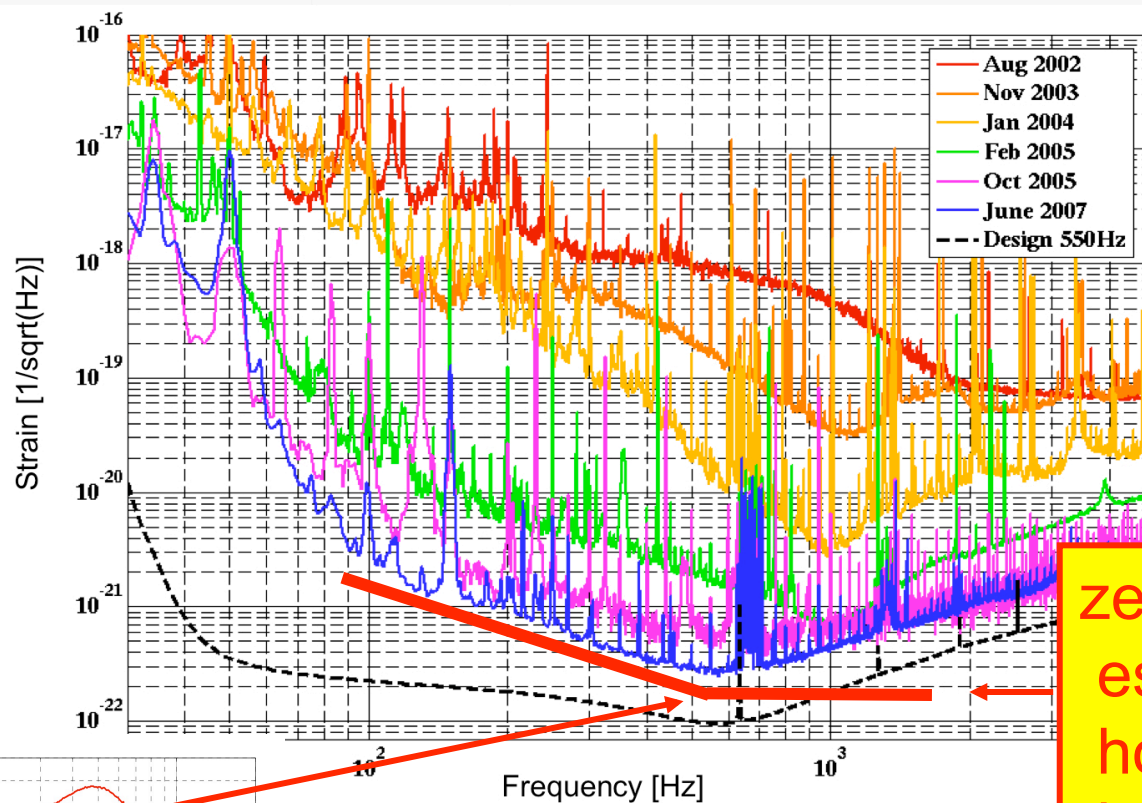


Noise in GEO600

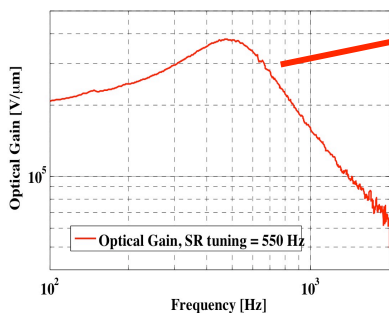


GEO Sensitivities

K.Strain



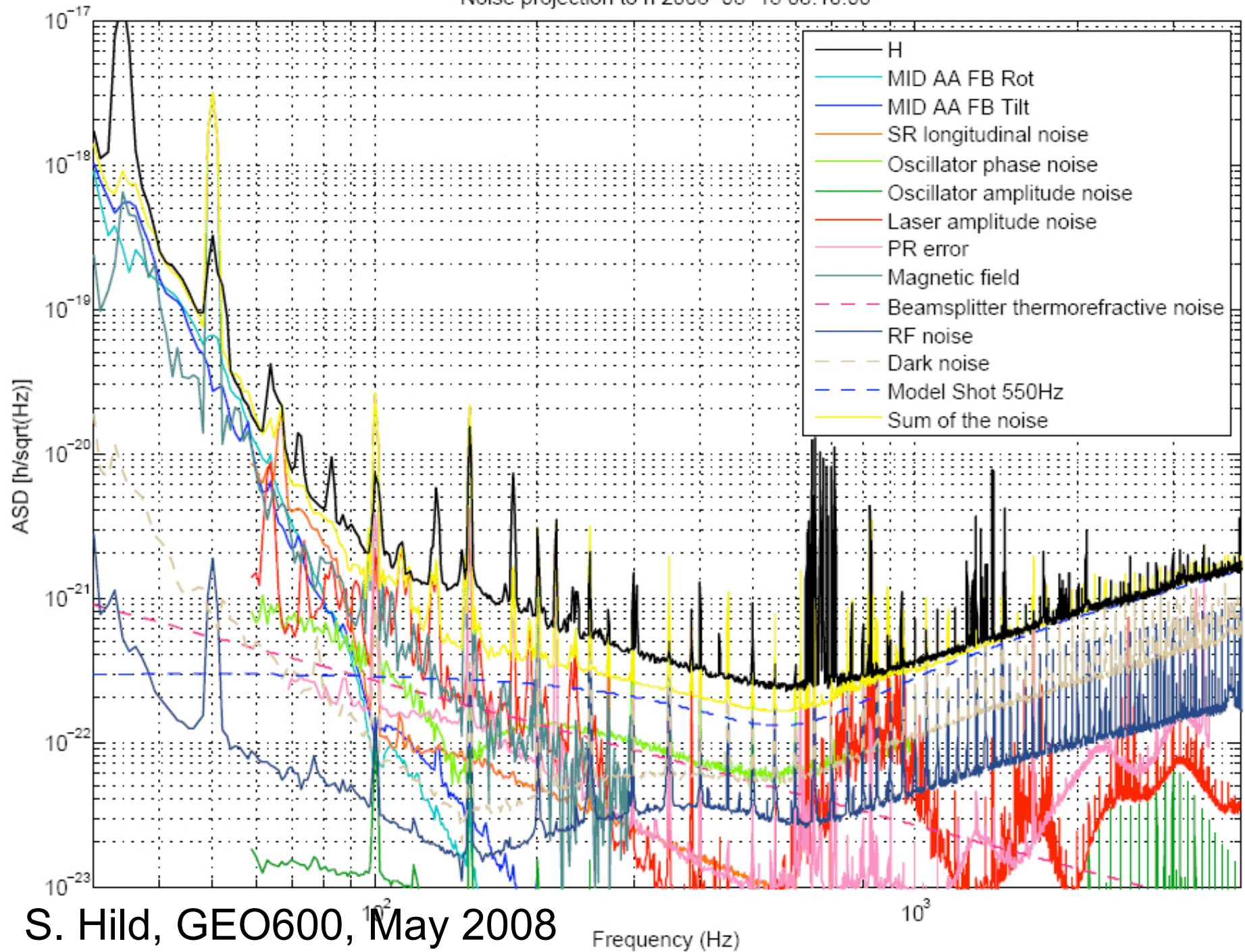
zero-parameter
estimate of
holographic noise
in GEO600
(equivalent strain)



H. Lück, S. Hild, K. Danzmann, K. Strain

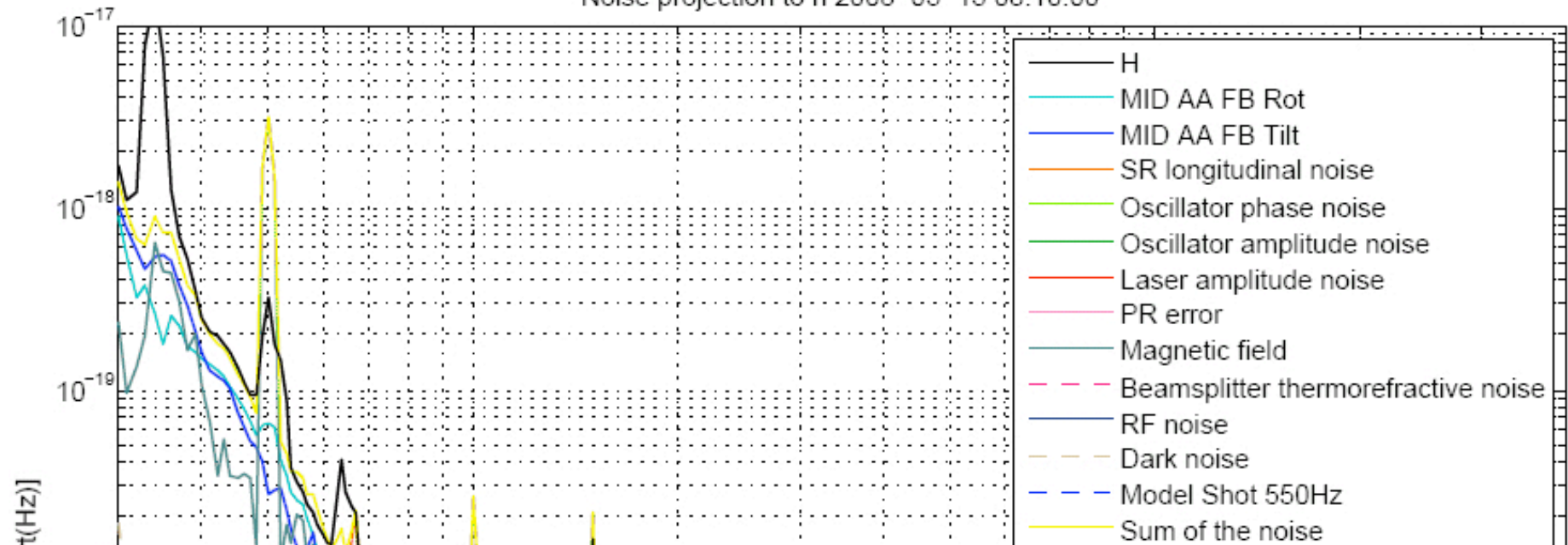
CJH: [arXiv:0806.0665](https://arxiv.org/abs/0806.0665)

Noise projection to h 2008-05-15 08:10:00

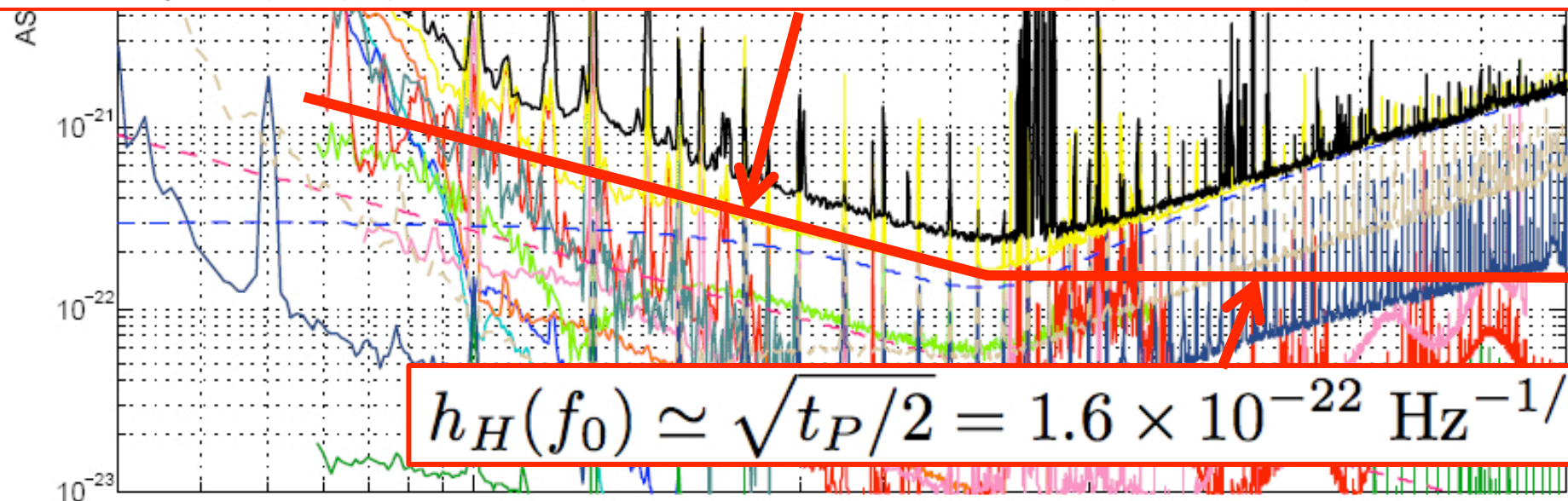


S. Hild, GEO600, May 2008

Noise projection to h 2008-05-15 08:10:00



$$h_H \simeq \sqrt{t_P/2} (f/550\text{Hz})^{-1} = 1.6 \times 10^{-22} (f/550\text{Hz})^{-1} \text{ Hz}^{-1/2}$$

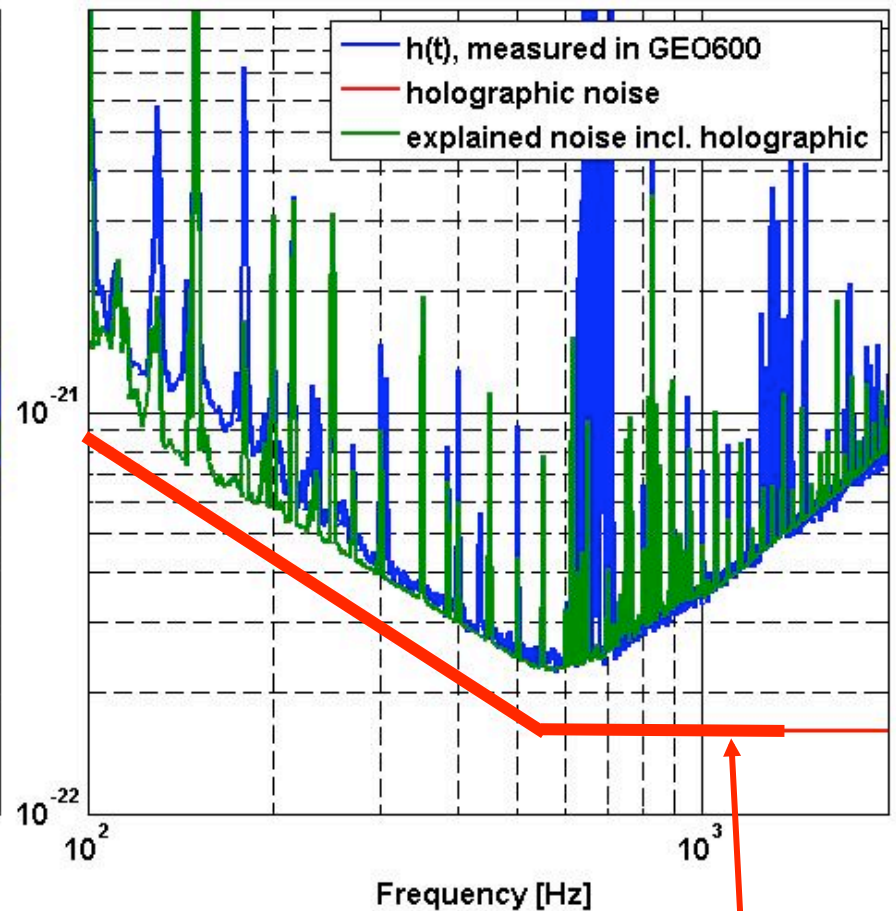
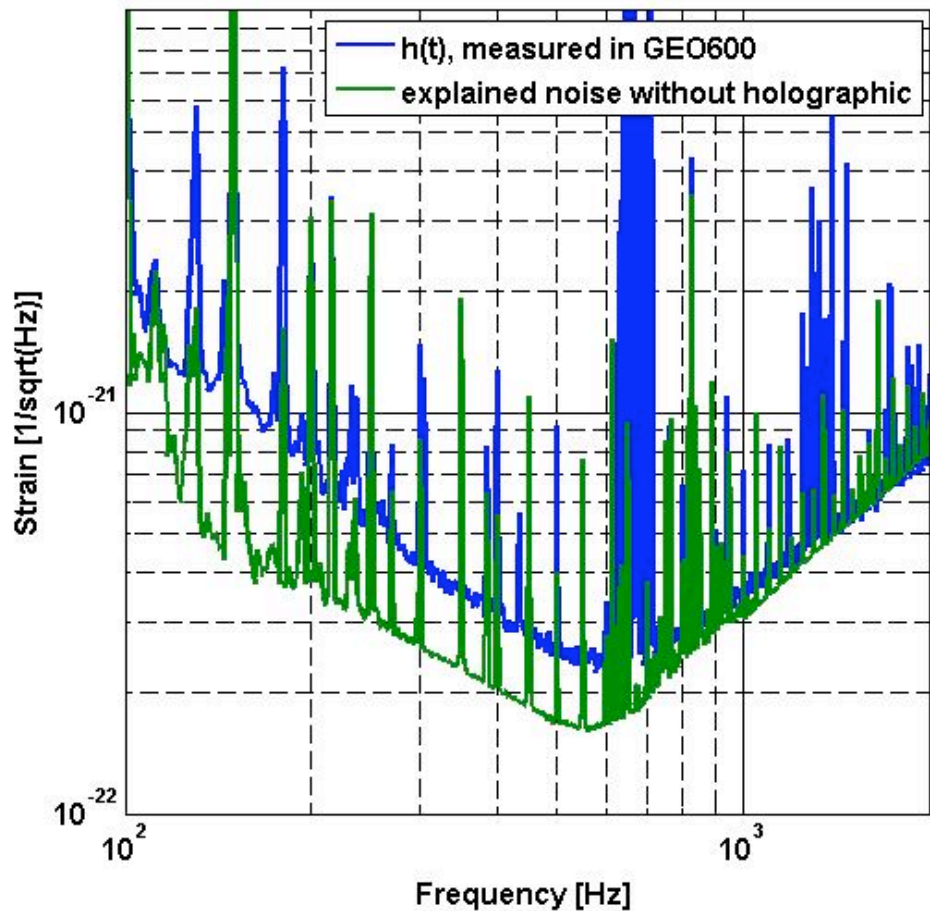


$$h_H(f_0) \simeq \sqrt{t_P/2} = 1.6 \times 10^{-22} \text{ Hz}^{-1/2}$$

S. Hild, GEO600

Frequency (Hz)

“Mystery Noise” in GEO600



Data: S. Hild (GEO600)

Prediction: [arXiv:0806.0665](https://arxiv.org/abs/0806.0665)

Total noise: not fitted

zero-parameter prediction
for holographic noise in
GEO600 (~ equivalent
strain)

Stefan Hild (U. Birmingham) and Harald Lück (Albert Einstein Institute), from the GEO600 team, will visit FNAL Dec 1-4, to work on the holographic noise interpretation of their data and share their expertise on the technology

Current experiments: summary

- Most sensitive device, GEO600, sees noise compatible with holographic spacetime indeterminacy
- other explanations, e.g. thermorefractive noise 3.6 times larger than expected in the model, need to be checked
- requires testing and confirmation!
- H. Lück: "...it is way too early to claim we might have seen something."
- But **GEO600 is operating at holographic noise limit**
- LIGO: current data not sensitive enough, awaits upgrade
- Models of holographic noise in signals of both systems can be improved

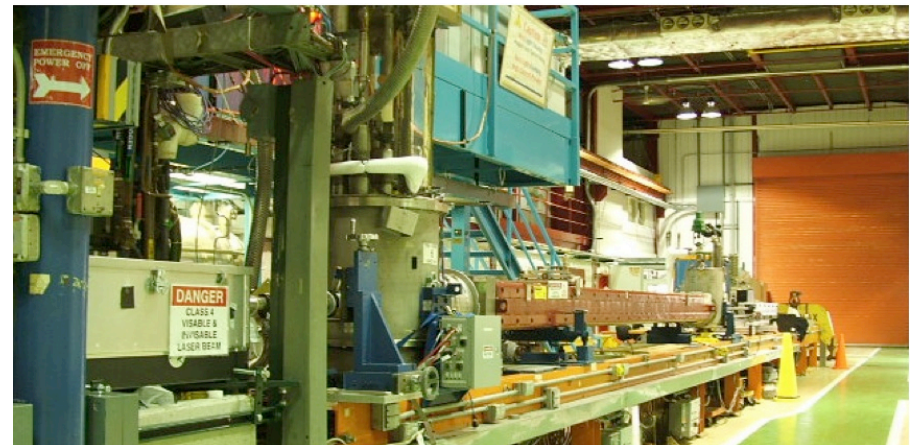
Interferometers can detect quantum indeterminacy of geometry

- Beamsplitter position indeterminacy inserts holographic noise into signal
- **system with LIGO, GEO600 technology can detect holographic noise if it exists**
- Signatures: spectrum, spatial shear

CJH: Phys. Rev. D 77, 104031 (2008); [arXiv:0806.0665](https://arxiv.org/abs/0806.0665)

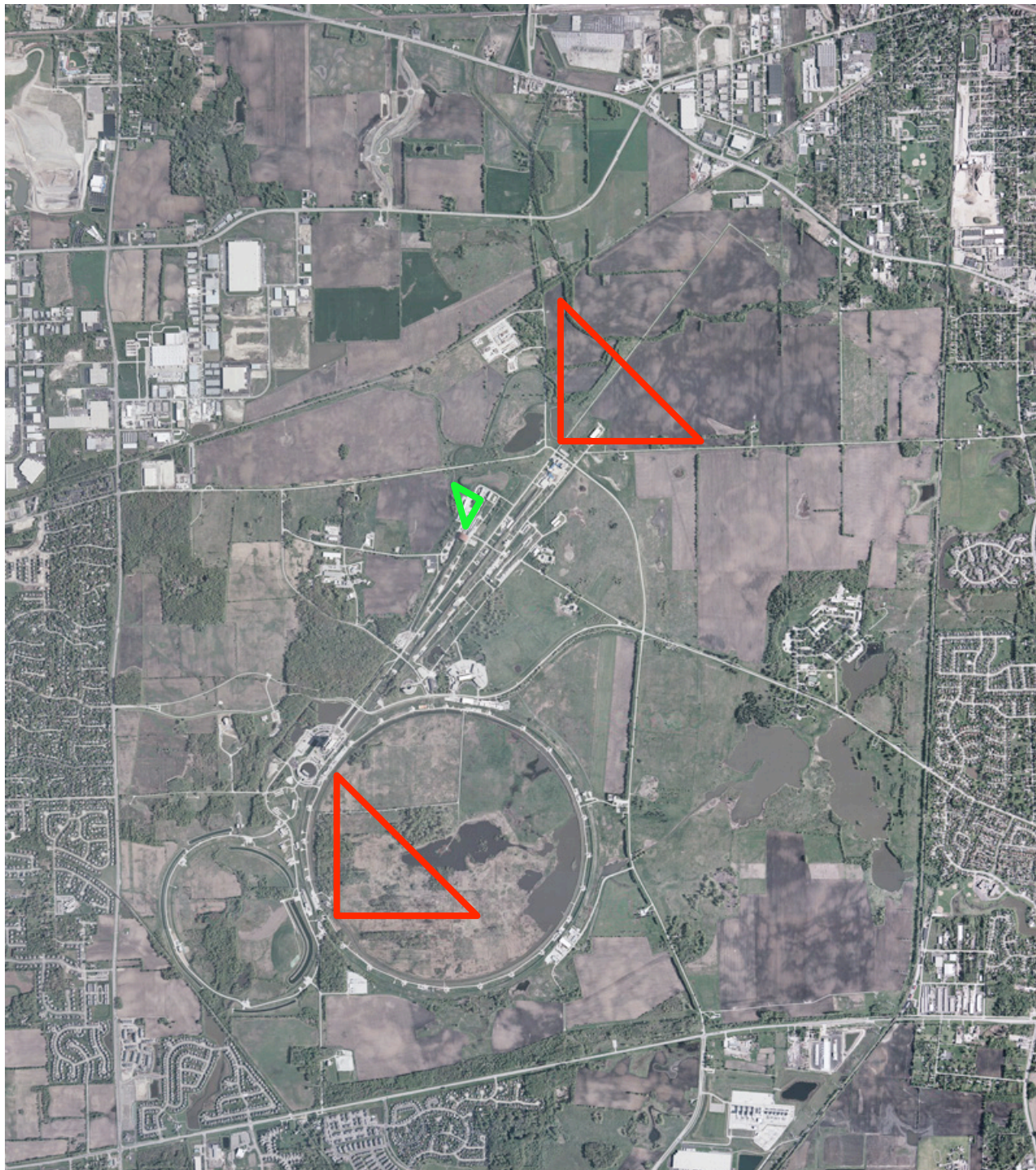
New experiments: beyond GW detectors

- Spectrum: $f \sim 100$ to 1000 Hz with existing apparatus
- $f \sim$ MHz possible with similar technique
- Correlated holographic noise in adjacent paths even at high f (LHO 2km with 4km? New setup?)
- Test geometry dependence, different configurations
- Optimal designs different from GW studies
- Shared technology with successor to FCPA's GammeV (A. Chou, W. Wester, et al.): cavities, lasers, detectors



~ 1 km arms
~100cm tubes
~100 kHz
~1kw cavity
or
~100m
~10cm
~1MHz
~100kW

for holographic
noise to be
detectable above
laser photon shot
noise



Experimental science of holographic noise

- Direct measurement of the fundamental minimum time interval
- Precisely measure Planck time: compare with value derived from Newton's G
- Measure quantum indeterminacy of spacetime, nonlocal quantum weirdness of spacetime metric
- Test predictions for spectrum and spatial correlations: holographic geometry, transverse position commutators
- Use quantum geometry in the lab to test interpretation of M theory: concrete physics of 2+1D null projection, black hole entropy, noncommuting position matrices
- Count all degrees of freedom up the Planck scale
- Seek clues to quantum physics of Dark Energy, inflationary fluctuations

Conceptual design for a “Quantum Time Machine”

- 2 interferometers, 100 m to 1 km
- Close but not too close ($\sim 10\%$)
- Sampling to above resonant frequency
- Correlate common holographic noise

Holographic quantum geometry: dark energy physics in the lab

- Holographic blurring is $\sim 0.1\text{mm}$ at the Hubble length
- $\sim (0.1\text{mm})^{-4}$ is the dark energy density
- “Nonlocality length” for dark energy is the same as holographic displacement uncertainty in the lab, scaled to Hubble length
- (literature on “holographic dark energy” centers on same numerology)
- Does not “explain” dark energy, but a big piece of the puzzle: quantum physics of empty space = 2+1D quantum theory

Next Steps

- Calculate the holographic-noise transfer function for GEO600 more precisely, with detailed model of signal recycling and detection
- Better model for Enhanced, Advanced LIGO predictions
- Survey experimental designs and trades at high frequencies
- Design a dedicated project for FNAL
- Survey other technologies for measuring high precision, low noise, nonlocal relative transverse positions (e.g., atom interferometers)
- Improve/axiomatize connections with M theory